Keywords: Data Processing, Grouping and Aggregation, Data Accumulation, Concept-oriented Model.

Abstract: Most of the currently existing query languages and data processing frameworks rely on one or another form of the group-by operation for data aggregation. In this paper, we critically analyze properties of this operation and describe its major drawbacks. We also describe an alternative approach to data aggregation based on accumulate functions and demonstrate how it can solve these problems. Based on this analysis, we argue that accumulate functions should be preferred to group-by as the main operation for data aggregation.

1 INTRODUCTION

1.1 Data Aggregation

If we ignore how data is organized in a data management system, then data processing of any kind can be reduced to computing new data values from the existing or previously computed values by applying some operations. These data processing operations can be broken into two major categories. Operations from the first category (which can be referred to as horizontal operations) process data values stored in individual elements like tuples, objects, documents or records which can be directly accessed from the source data element. Therefore, these operations are used for relatively simple record-level transformations as opposed to transformations based on subsets of records described below. The possibility to access all the necessary arguments of such an operation is based on some mechanism of connectivity. The dominant approach to connectivity relies on the relational join operation but other models and frameworks use references or links. The result of such operations is always computed from one or more attribute values of related elements (but not subsets). A typical example is computing the amount for an order item given its price and quantity attributes:

\[
\text{SELECT i.price \ast i.quantity AS amount FROM OrderItems AS i}
\]

A horizontal operation can be formally represented as a function of single-valued arguments. In the above example, such a function can be written as the expression \( \text{amount(i)} = \text{i.price} \ast \text{i.quantity} \). Here the output value is computed as a product of two input values which are accessed via attributes of this same record. In more complex cases, the arguments of the operation could be accessed using intermediate records but they still would be single values rather than subsets. For example, if the item price is stored in another table then this expression could be written as follows: \( \text{amount(i)} = \text{i.product.price} \ast \text{i.quantity} \). Most of the difficulties of this simple and natural approach are due to the connectivity mechanism which is responsible for providing access to related elements and data values (dot notation in the above example). In particular, the use of relational join makes it especially difficult (Savinov, 2016a) to directly apply such functional representation for computing new data values.

Operations from the second category (also referred to as vertical or aggregate operations) process data values stored in subsets of elements by aggregating multiple input values into one output value. These subsets, normally called groups, are represented and produced by the mechanism of grouping which defines how elements belong to one group depending on their properties. Grouping is as important for vertical operations as connectivity is for horizontal operations because they both determines what elements will be processed and how these elements will be accessed. The dominant approach to grouping assumes that all records having the same value of some attribute(s) belong to one group. For example, the total amount of one
order consisting of a number of order items can be computed using the following SQL query:

SELECT SUM(i.amount) AS total
FROM OrderItems AS i
GROUP BY i.order // Group definition

A vertical (aggregation) operation can also be formally represented as a function. However, this function takes set-valued arguments rather than single values in the case of horizontal operations. For example, in the above example this function is written as the following expression:

\[ \text{total(group)} = \sum \text{group} \]

where group is a subset of order items from the OrderItems table belonging to one order. Vertical operations are used for complex data processing and analysis. They are traditionally more difficult to understand and use than horizontal operations, and most of the difficulties are due to the grouping mechanism and user-defined aggregate functions.

1.2 Related Work

Relational algebra (Codd, 1970) is intended for manipulating relations, that is, it provides operations which take relations as input and produce a new relation as output. This formalism does not provide dedicated means for data aggregation just because it belongs to a set-oriented approach where the main unit is that of a set rather than a value. In particular, it is not obvious how to define and manipulate dynamically defined groups of tuples, that is, subsets which depend on values in other tuples. Theoretically, it could be done by introducing nested relations, complex objects and relation-valued attributes (see e.g. Abiteboul et al., 1989) but any such modification makes the model significantly more complicated and actually quite different from the original relational approach.

Since aggregation is obviously a highly important operation, these functions were introduced in early relational DBMSs and its support was added to SQL (Database Languages/SQL, 2003) in the form of a dedicated group-by operator. Importantly, group-by is not a formal part of the relational model but rather is a construct of a query language that supports this model but can also support other data models. In other words, group-by is not a specific feature of the relational model and actually does not rely on its main principles. In particular, it has been successfully implemented in many other models, query languages, database management systems and data processing frameworks. Nowadays, in the absence of other approaches, group-by is not merely a formal operation but rather a dominant pattern of thought for the concept of data aggregation.

An alternative approach to aggregation is based on using correlated queries where the inner query is parameterized by a value provided by the outer query. This parameter is interpreted as a group identifier so that the outer query iterates through all the groups while the inner query iterates through the group members by aggregating all of them into one value. Yet, this approach still needs the group-by operator but it is interesting from the conceptual point of view because it better separates different aspects used during data aggregation.

Aggregation is also a crucial part of the map-reduce data processing paradigm (Dean and Ghemawat, 2004) where map is a horizontal operation and reduce is a vertical operation. Its main advantage is that it allows for almost arbitrarily complex data processing scenarios due to the complete control over data aggregation and the natively supported mechanism of user-defined aggregations. Yet, map-reduce is much closer to programming than to data processing because manual loops are required with direct access to the data being processed.

Aggregation is an integral part of many other data processing frameworks like pandas (McKinney, 2010; McKinney, 2011), R or Spark SQL (Armbrust et al., 2015) which rely on data frames as the primary data structure. One of their specific features is that they provide a separate operation for grouping elements of a data frame so that different aggregate functions can be then applied to these groups as a next operation. This approach is also closer to programming models rather than to data models because user-defined aggregations still require direct access to and explicit loops through the group elements.

1.3 Goals and Contribution

This paper is devoted to the problem of data aggregation. We discuss this mechanism at logical level of data representation and processing, and not physical level where numerous implementations and optimization techniques exist taking into account various hardware architectures and network properties. The main mechanism for data aggregation which has been dominating among other approaches for dozens of years is the group-by operation. Yet, despite its wide adoption, this operation has some serious conceptual drawbacks. In particular, group-by does not naturally fit into the relational (set-oriented) setting and looks more like a
technical operator mixing various concerns and artificially attached to this rigor formalism. Group-by does not clearly separate such aspects as grouping (breaking a set into subsets) and aggregation (reducing a subset to one value). Also, it does not provide a principled mechanism for user-defined aggregations without support from the system level. The latter drawback – having no support for user-defined aggregate functions at logical level – makes this operation almost useless for complex data transformations and analytics. In other words, a data model without user-defined aggregations cannot be viewed as a complete general-purpose model. At least two fixes are possible and widely used in practice to overcome this problem, providing some support directly from the system or introducing explicit loops over the group elements for data aggregation. However, the former approach will turn the model into a physical one and the latter approach will turn it into a programming model. Therefore, the both currently existing general solutions are not acceptable if we want to have aggregation as an integral part of the logical model without any hooks to the physical level or extensions in the form of programming language constructs.

These significant drawbacks of group-by motivated us to rethink this mechanism and search for alternative approaches to data aggregation. As a possible solution to the existing problems of the group-by operation, we propose a new alternative approach based on accumulate functions which does not have these drawbacks and provides some significant benefits. The main idea behind this new approach is that an accumulate function incrementally updates the current aggregate as opposed to applying an aggregate function to a whole group by returning the final aggregate.

The standard approach to aggregation (Fig. 1a) means that we iterate through all the groups in the main loop, retrieve group members (facts) for each of them and then pass them as a subset to the aggregate function which returns a single value treated as the aggregate for this group. In the case of accumulate functions (Fig. 1b), we iterate through all the facts to be aggregated (not groups), pass each individual fact to the accumulate function (not a subset of facts) which updates the aggregate for the group element this fact belongs to.

The main benefit of accumulate functions is that they significantly simplify the task of describing data aggregation operations by clearly separating different concerns and factoring aggregation out into one small (accumulate) function. It does not require any support from the physical level of the model and uses only what is available at the logical level. It also fixes the problem of user-defined aggregations because accumulate functions do not loop through any subset but rather their purpose is to update the current aggregate using a new fact. In other words, describing the logic of aggregation is as easy as writing a normal arithmetic expression for defining new calculated columns.

<table>
<thead>
<tr>
<th>Groups</th>
<th>Facts</th>
</tr>
</thead>
<tbody>
<tr>
<td>a=SUM(g)</td>
<td>a=a+f.m</td>
</tr>
</tbody>
</table>

**Figure 1:** Aggregation (a) vs. accumulation (b).

The use of accumulate functions requires switching from the set-oriented paradigm to the function-oriented paradigm where a model is represented as a number of functions and data operations are described as function definitions (or expressions). We do not explicitly define such a function-oriented approach in this paper but familiarity with major principles of the functional data model (Kerschberg and Pacheco, 1976; Sibley and Kerschberg, 1977) could help in understanding how accumulation works. Accumulate functions have been implemented in the DataCommandr system (Savinov, 2016b) which uses the concept-oriented data model (Savinov, 2016c).

The paper has the following layout. In Section 2 we critically analyze properties of group-by by emphasizing some of its fundamental drawbacks in the context of data processing. Section 3 describes an alternative approach to data aggregation by introducing the mechanism of accumulate functions and discusses its advantages. Section 5 makes concluding remarks by summarizing drawbacks of group-by and advantages of accumulation.

## 2 “WHO IS TO BLAME?”
### GROUP-BY

#### 2.1 What is in a Group-By? Four Operations

Group-by operation takes one table, called *fact table*, as input and produces one table, called *group table*, as its output. One of the attributes of the fact table the values of which are aggregated is referred to as a
One attribute of the group table is computed during aggregation. For example, if OrderItems is a table consisting of order items (facts) each belonging to some order (group) then the group-by operator could be used to produce a list of all orders with large total amount computed from only cheap order items:

```
SELECT order, SUM(amount) AS total
FROM OrderItems
WHERE price < 10.0
GROUP BY order
HAVING total > 1000.0
```

Here we select only facts (order items from the OrderItems table) with low price (line 3). GROUP BY clause specifies that all facts from OrderItems having the same order attribute belong to one group (line 4). Once the facts have been filtered and grouped, the third step is to aggregate them. It is done by defining a new attribute in the SELECT clause as the standard SUM aggregate function (line 1). Finally, the set of groups is filtered by selecting only orders with large total amount (line 5).

Although group-by looks like one operation, it is actually a sequence of four components (Fig. 2) described below.

**Input filter** specifies criteria for selecting facts to be processed by group-by. These criteria are specified in the WHERE clause precisely as it is done in SELECT queries. All records which do not satisfy these criteria are ignored during aggregation. In the above example (line 3), we selected only cheap order items.

**Grouping criteria** provide conditions which determine a group each fact belongs to. In group-by, it is assumed that all facts that have identical values for the attributes listed after GROUP BY are assigned to one group. Implicitly, this means that each unique combination of values of the grouping attributes represents one group. In our example (line 4), all order items having identical order attribute belong to one group. The grouping component is responsible for the generation of the output table by instantiating its elements and in this sense it is an operation on sets because it takes a filtered input set and produced an output set.

**Aggregate function** converts measure values of all facts of one group to a single value which is then assigned to the aggregate attribute of the group table. An aggregate function is specified as a new attribute definition in the SELECT clause and a measure is specified as its argument. Thus the aggregation component is responsible for the generation of a new attribute and hence it is a column operation rather than a set operation. After executing this component, a new column storing aggregates for each group will be appended to the group table generated on the previous step.

**Output filter** is applied to the group table after aggregation and selects only groups which satisfy the criteria provided in the HAVING clause. In the previous example (line 5), we select only groups with the total amount greater than 1000.0.

```
FROM Group-by to Accumulation: Data Aggregation Revisited
```

Two of the four constituents of the group-by operation are filtering: the input fact table is filtered using WHERE clause and the output group table is filtered using HAVING clause. There is no clear reason why these two filters have been made integral part of the group-by operation. Theoretically, it could increase performance but a good query optimizer should not have any problems in producing the same query execution plan given input and output filters as separate operations. We treat such integration of input and output filters into group-by as one of its drawbacks because the operation becomes conceptually heavier and queries are getting more difficult to write and understand. Yet, in contrast to other drawbacks described below, it does not have significant consequences because these filters are essentially independent of the grouping component and the aggregation component, and therefore this drawback can be easily eliminated by simply not using these filters. In other words, removing the input and output filter steps from group-by will make this operation only better without any losses (except for maybe systems with no query optimization).

### 2.2 Grouping is Projection

Grouping operation is applied to a fact table and produces a new group table. Grouping criteria are specified using the *common value* semantics which means that facts having identical values for some attributes are considered related elements. In the case of group-by, they are considered members of one group which is (implicitly) identified by these values. In (Savinov, 2016a), we argued that the *common value* semantics is rather inappropriate for describing connectivity. It is also not very convenient for describing the group membership...
In particular, grouping criteria are a cross-cutting concern because each query has to repeat these conditions even if they are the same. It is difficult to modularize grouping definitions so that they can be reused in different queries. If we change the way groups are defined, then we need to update all queries. It is also difficult to specify grouping criteria using complex relationships derived from other data. For example, what if we want to group order items around their production place which however is derived from the data in other tables? A conceptually alternative approach to grouping consists in introducing some kind of ‘member_of’ relation for facts which determines a group each fact belongs to (Section 3.2).

How facts can be grouped is only one aspect of the grouping mechanism in group-by. A more serious problem is that conceptually, grouping as an operation has nothing to do with aggregation. In particular, it might well make sense to define one grouping to be used in many different aggregations or to apply one aggregation to many different groupings.

Just as for input-output filters, relational algebra already has such an operation. Indeed, grouping is essentially equivalent to the relational project where the result is a set of all distinct tuples composed of the specified attributes. Thus group-by provides its own internal version of relational project instead of reusing project as an independent operation that can be freely combined with aggregation and other operators.

In fact, grouping (as relational project) operation is not needed at all if the group table already exists. For example, if the database has already both the OrderItems fact table and the Orders group table then all groups as elements of the Orders table already exist and hence there is no need to do grouping at all. Yet, group-by does not allow us to skip the grouping step because it is an inherent part of group-by. The groups in this case will be built for each query execution with no possibility to reuse an already existing group table. For example, if we want to compute several aggregations for orders then it would be natural to reuse the existing Orders table or to produce it once and add the necessary columns with aggregates. Yet, in the case of group-by, each aggregated column will be produced along with the whole group table and all these individually generated group tables need to be joined if we want to have all aggregates in one output table.

Inclusion of grouping into group-by operator has the same consequences as including input-output filters into group-by. It is not possible to reuse one grouping operation for different aggregations which makes query writing more difficult and limits possibilities of optimization when translating such queries. However, in contrast to input-output filters which can be simply ignored or replaced by pre- or post-filters, grouping operation cannot be easily removed from group-by. The reason is that its parameters are needed in the aggregation operation and hence it is the aggregation component that has to be changed.

### 2.3 No User-defined Aggregate Functions

Complex data processing and analysis can hardly be done without custom aggregations performed by user-defined aggregate function as opposed to having only a limited set of standard functions like SUM. The standard group-by conception does not support user-defined aggregate functions and it is one of its biggest limitations. Yet, the problem is even worse because this mechanism is difficult to introduce without breaking some major principles of the relational model.

There are two major approaches to solving the problem of user-defined aggregate functions both of them being widely used in practice. The first approach introduces user-defined aggregate functions as extensions of the physical level of the system. They could be provided as external libraries which rely on this system API by essentially extending the set of standard functions. Once such a new aggregate function has been added, for example, by registering or linking its library, its name will be recognized by the query parser. This approach is used by many database management systems but we do not consider it in this paper because it does not change the principles of data aggregation at logical level of the model. In other words, at the logical level, the model and group-by still do not support custom aggregations and any system has its own support for these functions.

In the second approach, user-defined aggregate functions are provided at the level of the query language or the corresponding standard API. The most widespread technique consists in providing a possibility to define a normal function which gets a collection of elements or values as input and returns a single value as output. In particular, this simple and natural approach is the core stone of the map-reduce data processing paradigm where such functions are responsible for the reduce part of the data processing flow. For example, assume that we want to aggregate items for each order by using a
custom aggregate function which finds a weighted total (as opposed to a simple sum) by multiplying each order item amount by some factor which depends on this amount. The function iterates through all the group elements provided in the input collection and applies the necessary weight to each amount before updating the aggregate stored in the local variable:

\[
\text{DOUBLE weightedTotal(OrderItems group)} (1)
\]

\[
\begin{align*}
\text{DOUBLE out} &= 0.0; // \text{Aggregate} \\
\text{FOR(fact IN group) // Explicit loop} \\
\text{IF(fact.amount < 100)} \\
\text{out} &+= \text{fact.amount} \ast 1.0; \\
\text{ELSE} \\
\text{out} &+= \text{fact.amount} \ast 0.5; \\
\text{RETURN out; // Final value}
\end{align*}
\]

One problem with this approach is that it is actually not data processing anymore but rather a programming technique. In data modeling and data processing, in contrast to programming, the goal is to avoid explicit loops with intermediate state and direct access to the subset elements. A query writer is supposed to provide only criteria and options for data operations at the level of one element (instance). How the elements are iterated should not be defined in the query but rather is part of the system level (DBMS). This approach does not conform to general criteria for data modeling and data processing because such aggregate functions have to explicitly loop through the subset by maintaining an intermediate state between iterations in order to compute the aggregate. Essentially, the use of such type of user-defined aggregate functions, for instance in map-reduce, means switching to programming. Since such user-defined aggregate functions have a form of an arbitrary program, they cannot be easily integrated into a global query execution plan and have to be executed precisely as they are written.

Another problem with such user-defined aggregate functions is that they can be quite inefficient at run time for the following reasons:

- The system has to generate all the groups in an explicit form to pass them to the aggregate function. The problem is that there can be a huge number of such small groups or some groups could be very large (comparable with the complete data set).
- Computation of aggregates could be inefficient because it cannot be optimized for smaller and larger groups. Essentially, the user-provided code needs direct access to the data managed by the system which makes the task of physical data organization more difficult.
- Global optimization of different parts of a data flow can be difficult because they are written using non-compatible techniques and belong to different paradigms, for example, high level relational queries, custom reduce functions, and grouping provided by the system. For example, we could imagine the situation where a user-defined aggregate function needs not only data from this group but also data that results from some query by accessing other collections in this database.

Currently there are many sophisticated techniques that make data aggregation by means of user-defined aggregate function much more efficient especially taking into account properties of one or another underlying platform or data management system like Apache Hadoop or Spark (Zaharia et al., 2012). However, most of these approaches are being developed at physical level of the data processing system without any changes to the way aggregation is done at logical level and hence they treat aggregation as an extension rather than an integral part of the model. In this context, the goal is to get rid of explicit groups and explicit looping through these groups.

3 "WHAT IS TO BE DONE?"

3.1 ACCUMULATION

Accumulation is an alternative approach to data aggregation which is based on the functional data modeling paradigm as opposed to the set-oriented paradigm. The main difference is that an aggregated attribute is directly defined as a function which gets a single fact and knows only how to update the current value of the aggregate. It is referred to as an accumulate function because it is unaware of the whole group and does not know how to compute the final aggregate but rather knows only how to accumulate an individual fact into the intermediate aggregate.

To illustrate this difference, let us show how the standard \texttt{SUM} aggregate function can be equivalently defined as an accumulate function. Instead of looping through the group elements by returning the final aggregate as a sum of its values within one
procedure, we can only update the intermediate sum and return it:

\[
\text{SUM(OrderItems fact, DOUBLE out) \{ \\
    \text{RETURN out + fact.amount; \\
\}}
\]

Here the current value is passed in the out argument of the \text{SUM} accumulate function. Alternatively, the current value could be found from the fact:

\[
\text{SUM(OrderItems fact)} \{
    \text{Order group = fact.order; \\
    DOUBLE out = group.total; \\
    RETURN out + fact.amount; \\
\}
\]

Another variation of this approach is to directly update the value stored in the group table instead of returning it:

\[
\text{SUM(OrderItems fact)} \{
    \text{fact.order.total += fact.amount; \\
\}
\]

All the three modifications are conceptually equivalent because the only operation they do is modifying some intermediate aggregate using a new fact. The database engine loops through all the facts in the \text{OrderItems} table and passes each individual element to the accumulate function along with the current output of this same function for the group. This function updates the received output by adding the amount stored in the fact and returns the updated sum. This updated sum will be passed to this same method next time it is called for another fact of this same group. Here we also assume that either the system or the accumulate method itself can determine the group each fact belongs to.

Importantly, an accumulate function is essentially a column definition. Its name is the column name. Its (output value) type is the column type specifying what kind of data this column stores. Note that calculated columns are defined in the same way. For example, we could define a new calculated column as a function of two attributes:

\[
\text{DOUBLE discount(Order o) \{ \\
    \text{RETURN o.total + o.customer.bonus; \\
\}}
\]

The same could be written as a query:

\[
\text{CREATE COLUMN DOUBLE} \\
\text{o.discount = o.total + o.customer.bonus} \\
\text{TABLE Orders as o}
\]

The main difference of accumulated columns from normal calculated columns is that they use a different (fact) table to compute its output. Hence, they need an additional parameter with the fact table name. For example, a column with the total amount for all orders could be defined as follows (Fig. 3):

\[
\text{CREATE COLUMN DOUBLE \\
o.total = o.total + i.amount} \\
\text{FACT TABLE OrderItems i} \\
\text{GROUP PATH i.order}
\]

In addition to the \text{DOUBLE} column type, we specify its name as a function name (\text{total}). Actually, it can be any function that returns a double value and is defined elsewhere or in-line (like the \text{SUM} accumulate function). The \text{TABLE} keyword is followed by the table name this column belongs to. In this example, we define a new accumulated column for the \text{Orders} table. The \text{FACT TABLE} keyword is the most important one because it changes the way this column is computed. It says that the \text{total()} function will be evaluated for each element of the \text{OrderItems} table rather than for the \text{Orders} table where this column is defined. Without this keyword, it will be evaluated for each element of the \text{Orders} table and will be a normal calculated column like \text{i.price*i.quantity AS amount} from Section 1.1.

![Figure 3: Structure of the accumulation.](image)

This generic approach has several minor design alternatives which do not change its essence but can change its flexibility and performance under certain conditions. The alternatives depend on which part – the query or accumulate function – is responsible for and aware of the following aspects:

- **Grouping** – how to determine a group element the current fact belongs to. In the above example, we assumed that grouping is specified at the level of the query in the \text{GROUP PATH} clause. In the next subsection we will discuss other alternatives and properties of the grouping mechanism.

- **Measures** – what attributes of the facts have to be aggregated. In our examples, we assumed that an accumulate function gets a fact as an element (tupple). Other alternatives and properties of user-
defined accumulate functions are discussed in Section 3.3.

3.2 Grouping is Membership Relation

The mechanism of accumulation does not use a separate grouping step based on the relational project operation as it is done in group-by. Instead, accumulation relies on the group relation between the fact table and the group table. The main task of the group relation is to determine a group each individual fact belongs to and it can be any function from the group table to the fact table. For example, if we want to view orders as groups consisting from order items then the order attribute of the OrderItems table will represent a group relation. Given an order item, we can always get its group as output of this attribute. Importantly, we do not perform projection or computation of some new table (although formally it can be done) because relational project is not used in the accumulation mechanism.

Since a group relation is a normal function, we do not introduce a new mechanism but simply use a new semantics of links (Savinov, 2016a, Section 3.1): a link points to an element (group) this element (fact) belongs to. One advantage of this approach is that grouping is actually defined at the level of the whole model and the existing group relations can be reused in aggregations as well as other operations like project. Moreover, a group relation can be an arbitrary function which actually computes its output so that groups are derived dynamically during computations. It is always possible to provide custom or user-defined groups instead of using only table attributes for representing the group relation.

Another property of this grouping mechanism is that both the fact table and the group table are assumed to already exist, that is, no new table results from the accumulation. If the group table does not exist, then it has to be created. In particular, it can be produced from the fact table by using projection along the group relation.

One design alternative in the context of group relation is whether it is provided in the query itself or encoded into the accumulate function. If the group (relation) path is specified at the level of the query (like GROUP PATH in query (4)) then the query engine will use it to retrieve a group for each fact, get the current aggregate for this group element and pass this value to the accumulate function which will return an updated aggregate for this group. Here we explicitly declare the group relation at the level of the query and then the engine uses it for aggregation. An alternative approach is where all these steps are performed by the accumulate function itself and hence the group relation is not declared – it is simply part of the logic of each accumulate function. Query (4) can be rewritten without an explicit group path which instead is used in the accumulate function:

```sql
CREATE COLUMN DOUBLE o.total = 1.order.total + 1.amount
TABLE Orders o
FACT TABLE OrderItems i
GROUP PATH i.order // Not used
```

This query knows only that the total function has to be evaluated for each element in the OrderItems table and hence an accumulate function will take only one parameter – an element of the fact table. However, the accumulate function has to determine the group and also retrieve its current aggregate (underlined fragment in this example). Such queries are somewhat simpler and this approach could be more flexible in some situations. However, it is necessary to understand that such accumulate functions mix two concerns, grouping and aggregation. Also, the system could define groups for the facts more efficiently by applying the necessary optimizations which is not possible if grouping is done by the accumulate function.

3.3 User-defined Accumulate Functions

In group-by, user-defined aggregate functions are either not supported or require writing an explicit loop by getting a collection of elements as a parameter. In contrast, an accumulate function processes one instance rather than a collection by receiving a single (fact) element as input and returning a single (updated) value as output. For example, if we want to compute weighted total then instead of the aggregate function (1) in Section 2.3 we can write the following accumulate function:

```java
DOUBLE weightedTotal(OrderItems fact) {
    IF(fact.amount < 100)
        RETURN out + fact.amount * 1.0;
    ELSE
        RETURN out + fact.amount * 0.5;
}
```

This function receives the current value via the out argument (or using the function name) and updates it by adding the measure amount weighted by some factor depending on its value. Thus this function updates its previous output by assuming that it will be called many times for each element of one group.
Accumulate functions define new columns using function operations rather than new sets using set operations. Therefore, they can be easily used to define new columns very similar to how calculated columns are defined. For the same reason, accumulate functions cannot be directly used in a conventional data flow graph where nodes are sets and edges are set operations. Actually, the attempt to make aggregation integral part of a set algebra is precisely why the group-by is so conceptually eclectic and looks more like an artificial addition to some framework than an independent formal operation. And treating aggregation as a column definition is precisely why accumulate functions are operation. And treating aggregation as a column definition is precisely why accumulate functions are so simple and natural.

An accumulate function can be viewed as an aggregate function where both the loop and the local variable with the current aggregate are factored out of the procedure. The accumulate function itself defines only the body of this loop without any intermediate state. This makes the whole approach not only conceptually simpler but also potentially more efficient because organizing and optimizing various processing loops is precisely what a database engine is intended for.

Our examples assume that an accumulate function gets one fact element as a parameter. This fact is then used to update the current output. In this case, the measure is not explicitly declared at the query level and it is not known which property of the fact will be actually used to update the current aggregate. Thus this approach does not limit aggregation by only one measure attribute. It is possible to use other attributes, functions or other data elements that can be accessed from this fact element. For example, we could compute the total for each order by using order item price and quantity directly from the accumulate function:

```
CREATE COLUMN DOUBLE o.total = o.total + i.price*i.quantity
TABLE Orders o
FACT TABLE OrderItems i
GROUP PATH i.order
```

Here the `measure` keyword denotes the value that has to be used to update the current aggregate and is used instead of the whole fact reference. This approach is less flexible from the point of view of aggregation but provides more possibilities for query optimization because the engine knows more about what data the accumulate function will use.

It is important to understand that the semantics of accumulate functions is different from that of aggregate functions. If we want it to be equivalent to an aggregate function implemented by explicitly looping through a group then it has to satisfy certain formal criteria. First, the result has to be independent of the order of updates and hence this operation has to obey the commutative law: \( a + b = b + a \). Second, to be able to apply updates to partial results, an accumulate function has to obey the associative law: \((a + b) + c = a + (b + c)\).

Implementing the traditional logic of aggregation using accumulate functions is easy for some functions like `SUM` or `MAX` but it can be unobvious for other functions. For example, computing an average value is reduced to defining two accumulate functions, `SUM` and `COUNT`, while the result is a new calculated function dividing the sum by the count for each row. Expressing the logic of aggregation using accumulate functions can be rather tricky, for example, if we want to compute a median value. However, it is necessary to understand that the conventional aggregate functions will always be more expressive and more flexible just because we essentially get full control over the computation process. Yet, we do not treat it as a drawback because it as a natural (and typical) limitation caused by delegating some functionality (loop organization and optimization) to the system.

Accumulate functions are used to define columns and hence they cannot be easily integrated into a set-oriented model or data processing system. A data model with accumulate functions has to support functions and function operations. In other words, the conventional approach to data processing consists in defining a graph where nodes are sets and edges are set operations. In contrast, accumulate functions require a model where nodes can be functions (columns) and edges are function operations. In such a model, every function is defined in terms of other functions and evaluating a function means finding its output values given output values of the functions it depends on. Apparently, this approach cardinally differs from the set-oriented paradigm. The principles of such a
model were described in (Savinov, 2016c) and implemented in the DataCommandr system (Savinov, 2016b).

4 CONCLUSIONS

Although group-by is a powerful data processing operator, it has the following conceptual drawbacks:
- Group-by is an eclectic mixture of several quite different operations rather than one operator
- Grouping in group-by is essentially the relational project operation which conceptually has nothing to do with aggregation
- Group-by does not inherently support user-defined aggregate functions without explicit loops and without system support

We also proposed and analyzed an alternative mechanism to data aggregation based on accumulate (update) functions which provides the following benefits:
- The complete logic of aggregation is modularized in one accumulate function which defines a column in terms of other columns
- Accumulation uses group membership relation (which is also a function) and does not involve relational project
- Accumulation inherently supports user-defined functions because it is a functional approach which is based on column operations rather than set operations

Taking into account these properties we argue that the mechanism of accumulation should be preferred to group-by as the main aggregation operation in data models and data processing frameworks. However, it belongs to the functional data modeling paradigm which is based on defining and manipulating functions rather than sets. Formal integration of the functional and set-oriented approaches including set operations as well as (horizontal and vertical) column operations will be our focus for future research.

REFERENCES

Savinov, A., 2016a. Joins vs. Links or Relational Join Considered Harmful. Internet of Things and Big Data (IoTBD’2016), 362–368