

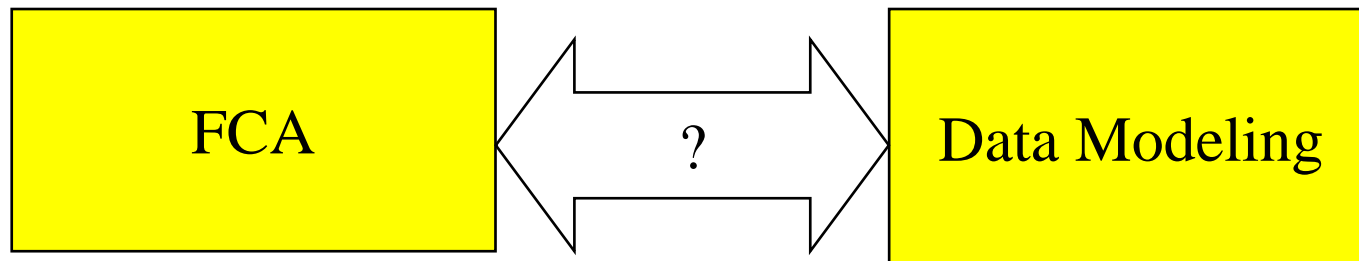


Hierarchical Multidimensional Modelling in the Concept-Oriented Data Model

Alexandr Savinov
Fraunhofer Institute for Autonomous Intelligent Systems
Knowledge Discovery Team
Germany
alexandr.savinov@ais.fraunhofer.de

Contents

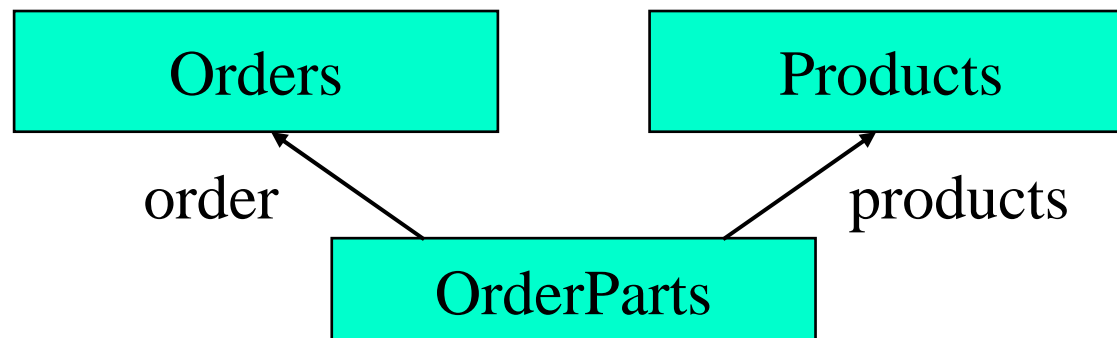
- Introduction
- Logical and Physical Structure
- Dimensions and Inverse Dimensions
- Projection and De-projection
- Multidimensional Grouping and Aggregation
- Conclusion



Introduction

Data models and dimensionality modelling

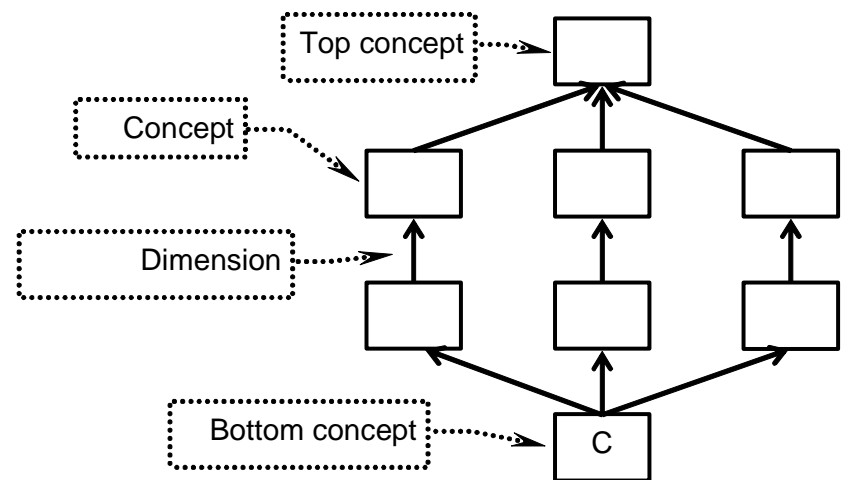
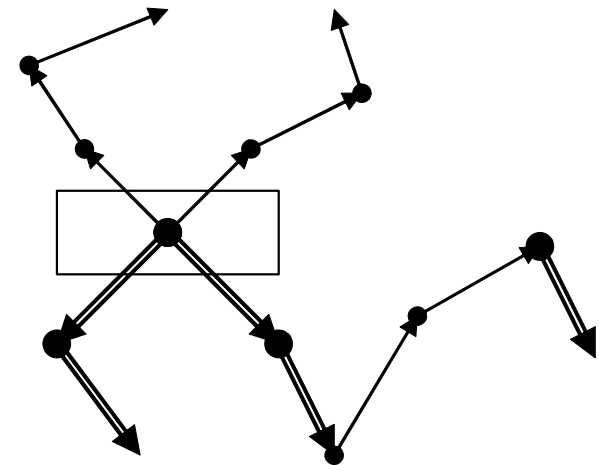
- Entities and relationships (ERM)
 - Logic and predicates (deductive databases)
 - Relations (RM)
 - Facts (ORM)
 - Objects (OODM)
 - Dimensions (OLAP, multidimensional databases)
-
- Dimension is a named link between subconcept and superconcept



Introduction

Assumptions and related work

- Global semantics (URM)
- Using the structure for navigation (FDM)
- Hierarchical structure (FCA)
- Level of details (OLAP)



Introduction

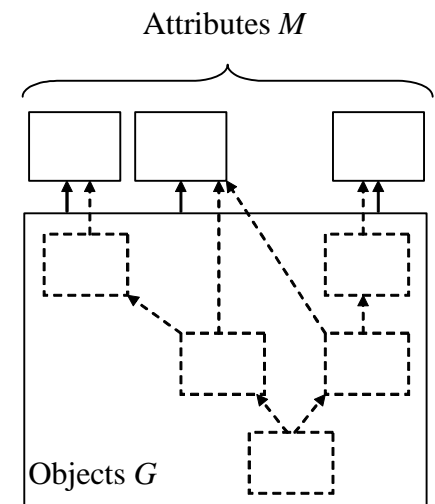
FCA

- Concept -> Concept
- Object -> Data item
- Attribute -> Primitive concept

- In FCA concepts depend on data while in COM data depends on concepts, that is concepts define a structure for data (in FCA the structure is derived from data semantics)

- Items belong to one concept while in FCA object may belong to many concepts

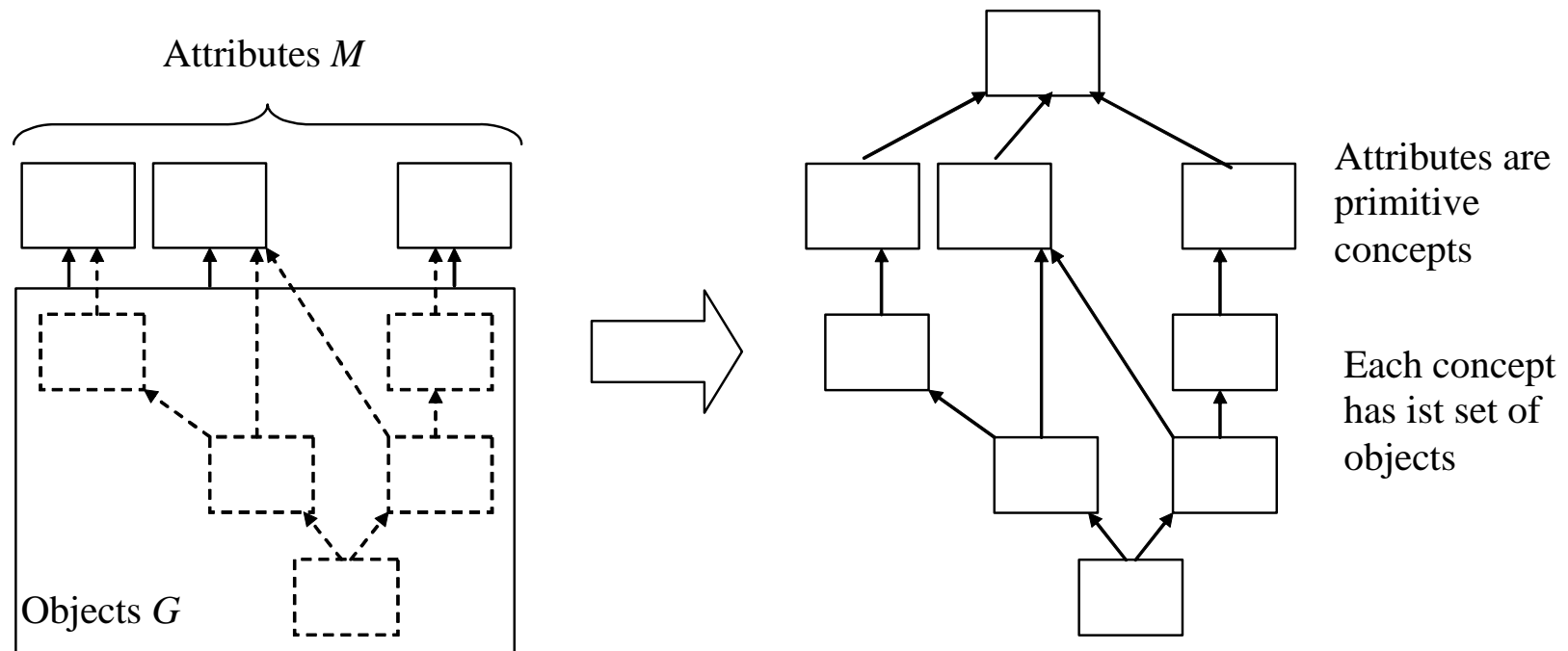
- COM concept is a (non-primitive) attribute for subconcepts



Introduction

Questions

- Why we have (primitive) attributes defined at structural level while concepts are derived from data semantics?
Why not to have a possibility to define a (non-primitive) attribute as a concept?

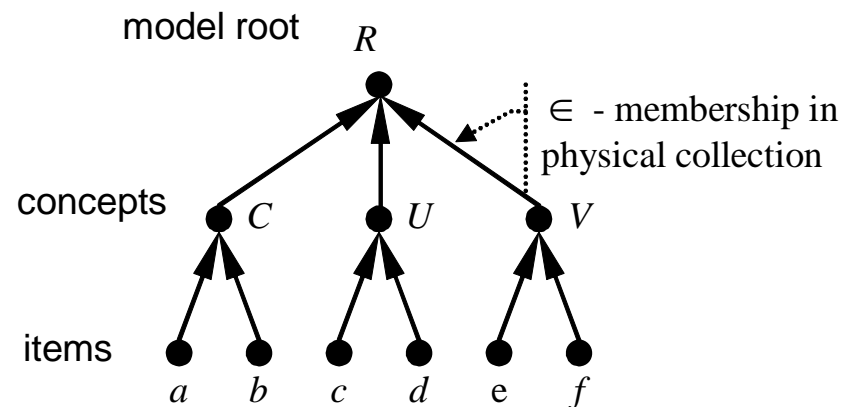


Physical and Logical Structure

Physical structure

- At physical level an element of the model is a collection of other elements
- Physical structure is used for representation and access
- Physical structure is used to implement reference
- Physical structure is hierarchical where each element has only one parent

$C = \{a, b, \dots\}$
 $a \in A, b \in A, \dots$

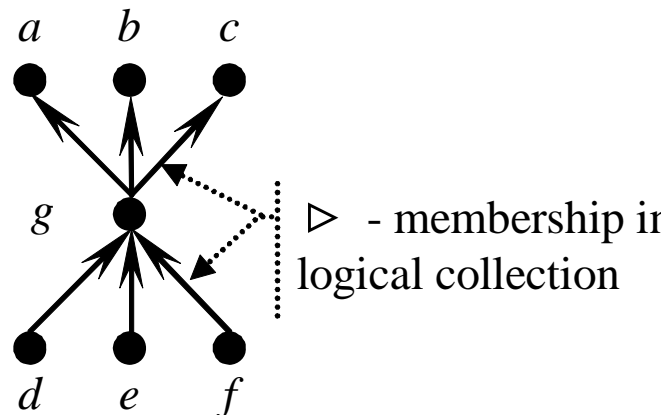


Physical and Logical Structure

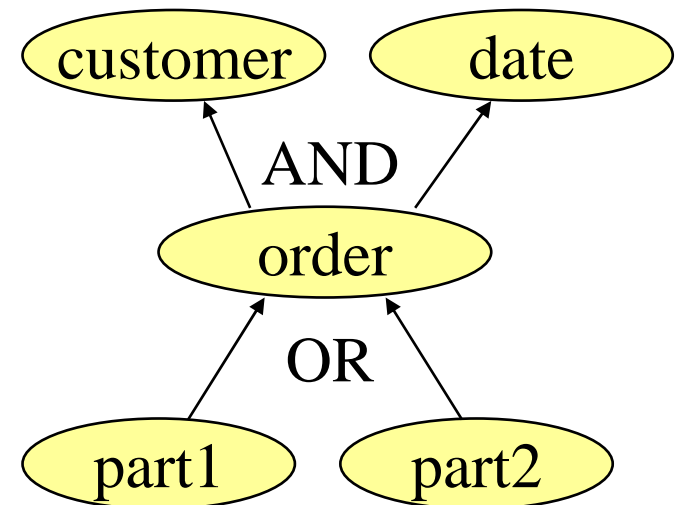
Logical structure

- Each element is a combination of other elements (by reference)
- Logical structure is used to represent data semantics (properties)
- Logical collection is a dual combination
- Each element has many parents and many children

$$g = \langle a, b, \dots, c \rangle, a \triangleleft g, b \triangleleft g, \dots, c \triangleleft g$$



$$g = \{d, e, \dots, f\}, d \triangleright g, e \triangleright g, \dots, f \triangleright g$$



Physical and Logical Structure

Two level model

- **[Root]** One root element R is a physical collection of concepts, $R = \{C_1, C_2, \dots, C_N\}$
- **[Syntax]** Each concept is
 - (i) a combination of other concepts called *superconcepts* (while this concept is a *subconcept*), $C = \langle C_1, C_2, \dots, C_n \rangle \in R$
 - (ii) a physical collection of *data items* (or concept instances), $C = \{i_1, i_2, \dots\} \in R$
- **[Semantics]** Each data item is
 - (i) a combination of other data items called *superitems* (while this item is a *subitem*), $i = \langle i_1, i_2, \dots, i_n \rangle \in C$
 - (ii) empty physical collection, $i = \{\}$

Physical and Logical Structure

Two level model

- **[Special elements]** If a concept does not have a superconcept then it is referred to as *primitive* and its superconcept is one common *top concept*, and if a concept does not have a subconcept then it is assumed to be one common *bottom concept*, and an absence of superitem is denoted by one special *null item*.
- **[Cycles]** Cycles in subconcept-superconcept relation and subitem-superitem relation are not allowed,
- **[Syntactic constraints]** Each data item from a concept may combine only items from its superconcepts.

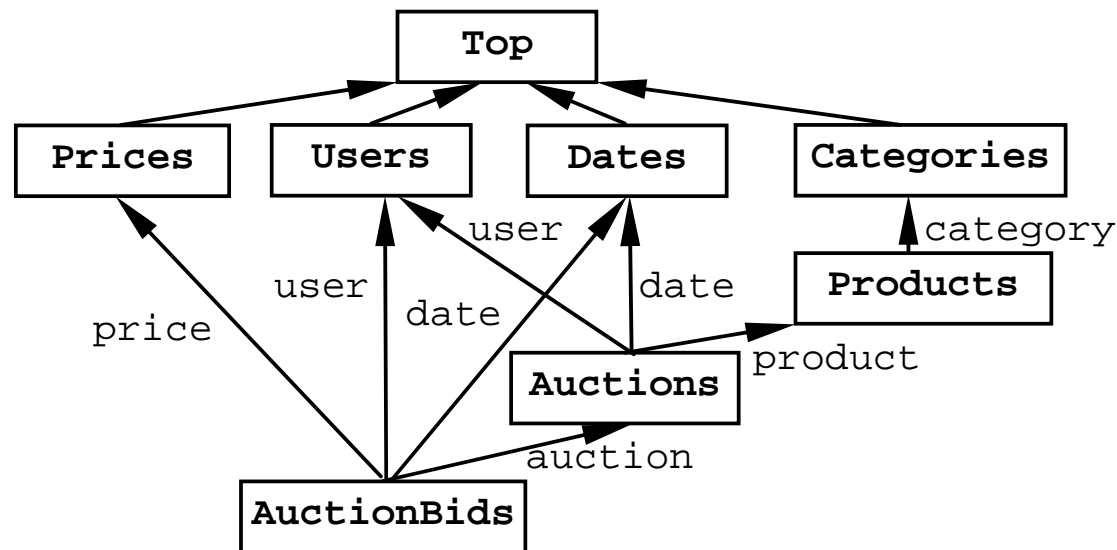
Syntax and Semantics

Model syntax

- At syntactic level a concept is a combination of its superconcepts

$$C = \langle x_1 : C_1, x_2 : C_2, \dots, x_n : C_n \rangle$$

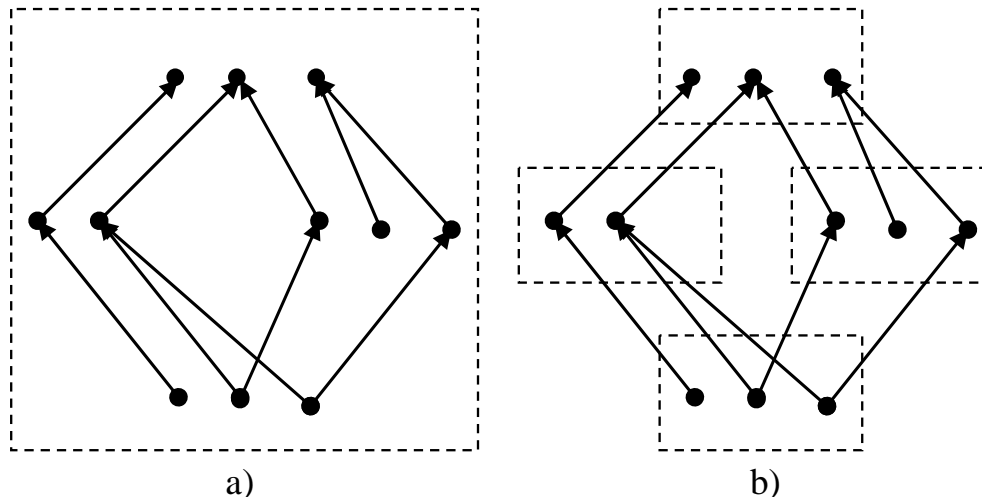
- Each superconcept is identified by dimension name, that is, dimension is a relative position of superconcept



Syntax and Semantics

Model semantics

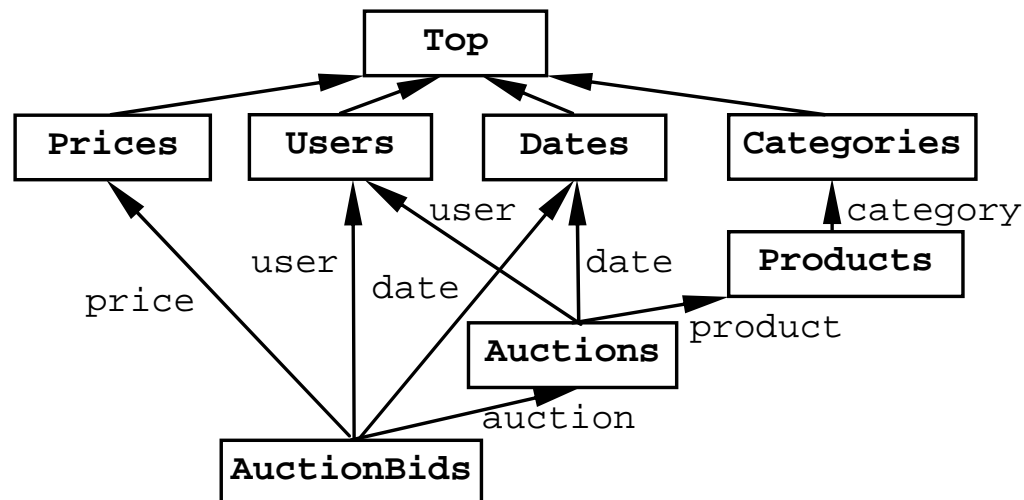
- Each concept is a set of items: $C = \{i_1, i_2, \dots\}$
- An item is a combination of its superitems: $i = \langle i_1, i_2, \dots, i_n \rangle$
- There is no difference between objects and attribute values: an object has values in other objects, and it is a value for other objects



Model Dimensionality

Dimensions

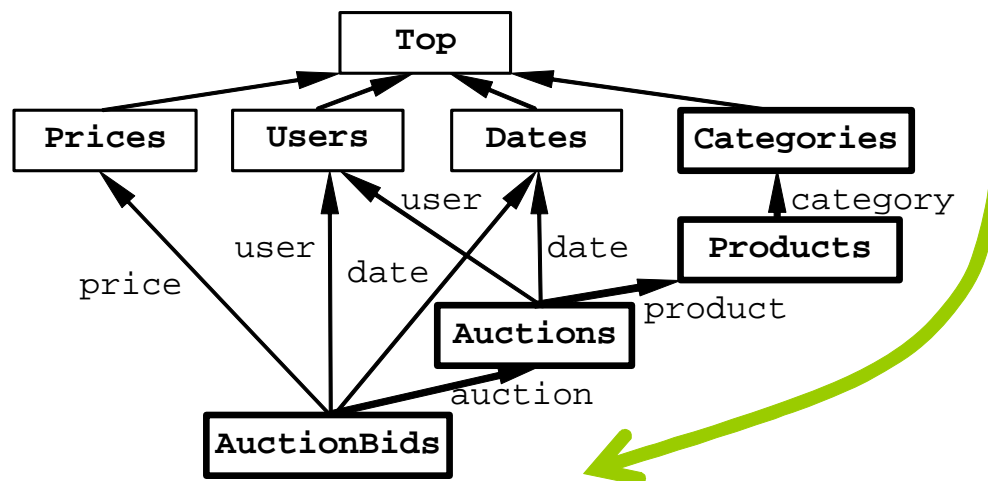
- Dimension is a named position of superconcept
- Superconcept is referred to as the domain
- Dimensions of higher rank consists of many (local) dimensions
- Dimension with the domain in a primitive concept is a *primitive dimension*
- The number of primitive dimensions is the model *primitive dimensionality*



Model Dimensionality

Inverse dimensions

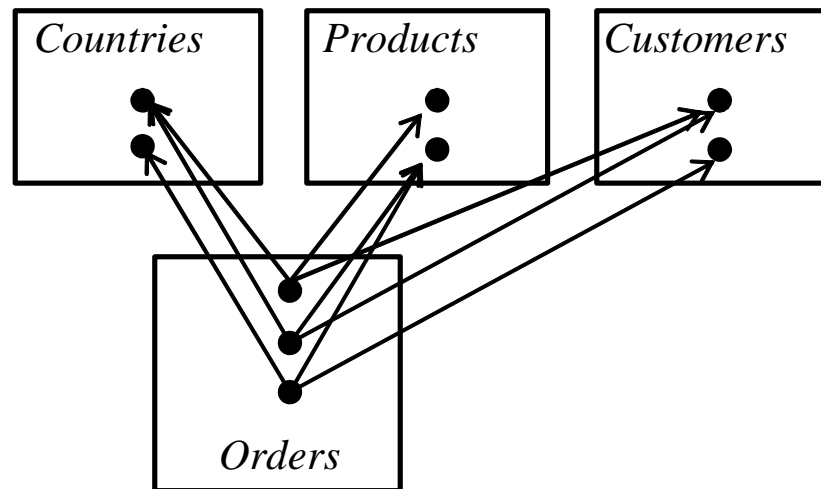
- Inverse dimension has an opposite direction
- Inverse dimension identifies a subconcept
- Inverse dimensions are multi-valued (while dimensions are one-valued)
- The number of primitive dimensions is equal to the number of primitive inverse dimensions
- {AuctionBids.auction.product.category}



Model Dimensionality

Logical collections

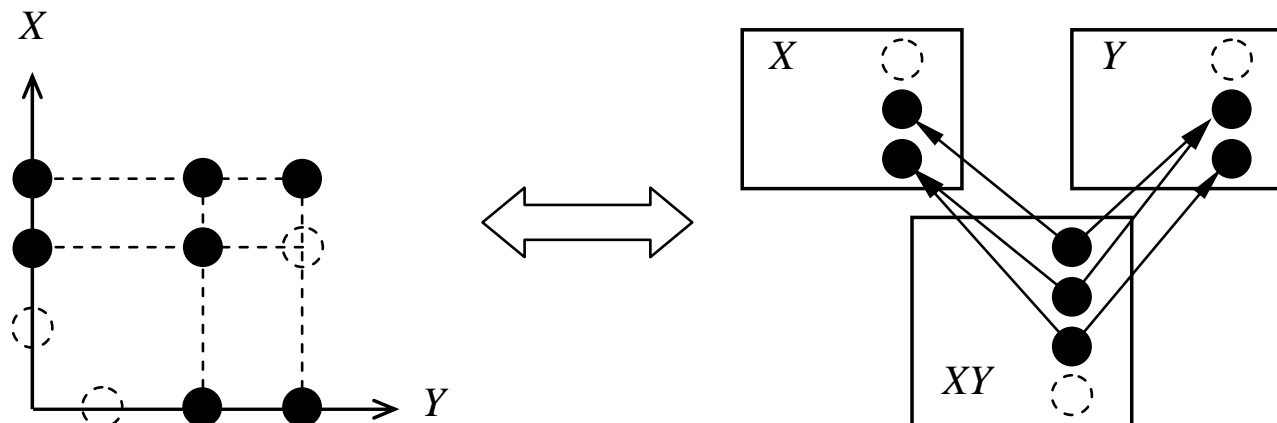
- A concept is a logical collection of its subconcepts
- An item is logical collection of its subitems
- An item is group for its subitems



Model Dimensionality

Hierarchical coordinate system

- A concept can be interpreted as an axis with items as coordinates
- A coordinate has its own coordinates and points can be used as coordinates for other points



Projection and De-projection

Projection

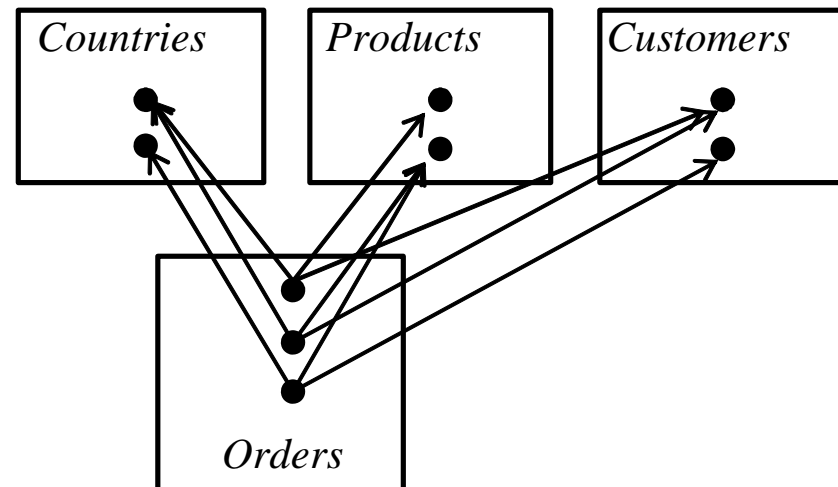
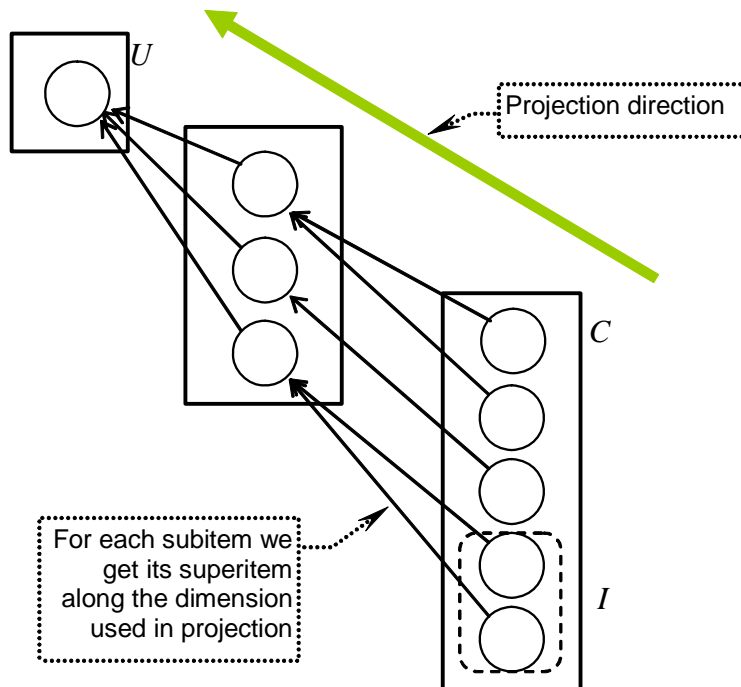
- Projection of a subset of subitems along some dimension path:

$$I \rightarrow d = \{u \in U \mid i.d = u, i \in I \subseteq C\}$$

$$I \subseteq C$$

$$d = d^1 . d^2 . \dots . d^k$$

$$U = \text{Dom}(d)$$



Projection and De-projection

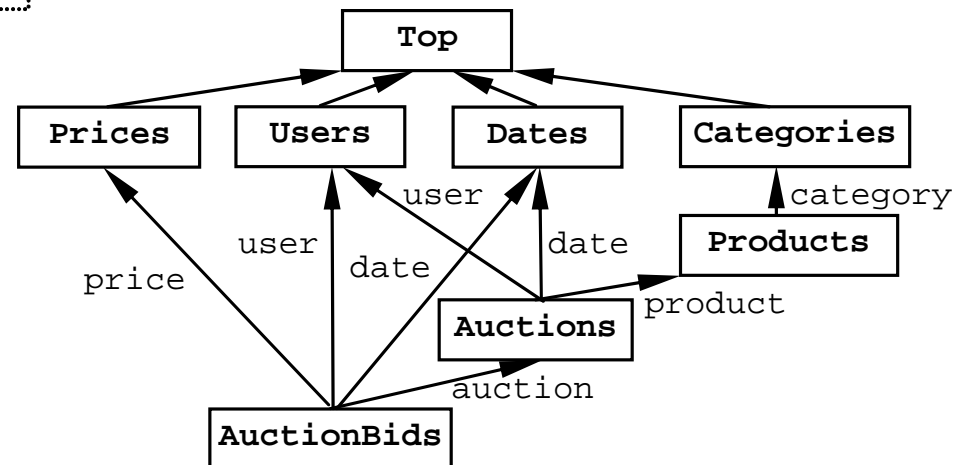
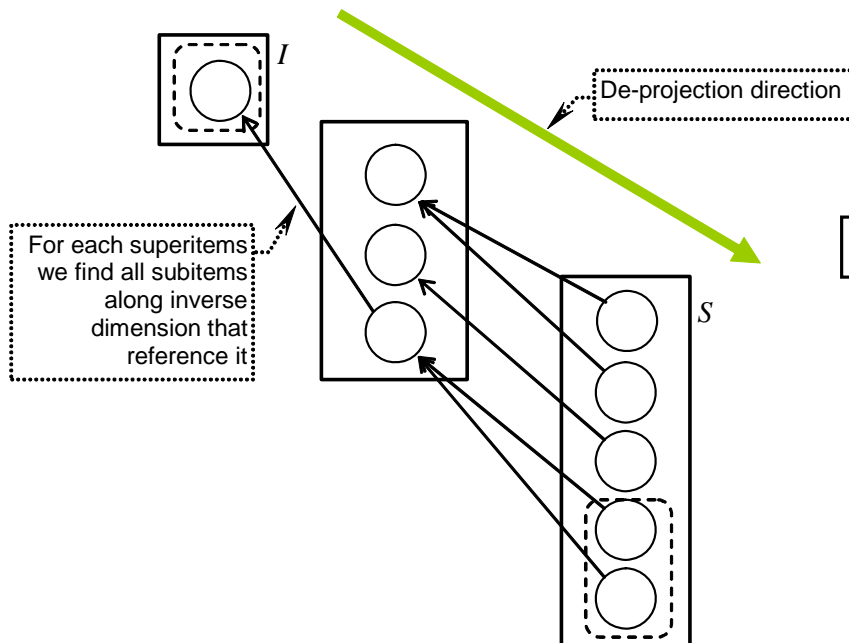
De-projection

- De-projection of a subset of superitems along some inverse dimension:

$$I \rightarrow \{d\} = \{s \in S \mid s.d = i, i \in I \subseteq C\}$$

$$\{d\} = \{d^1.d^2.\dots.d^k\}$$

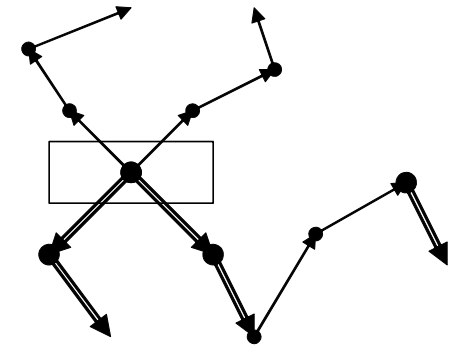
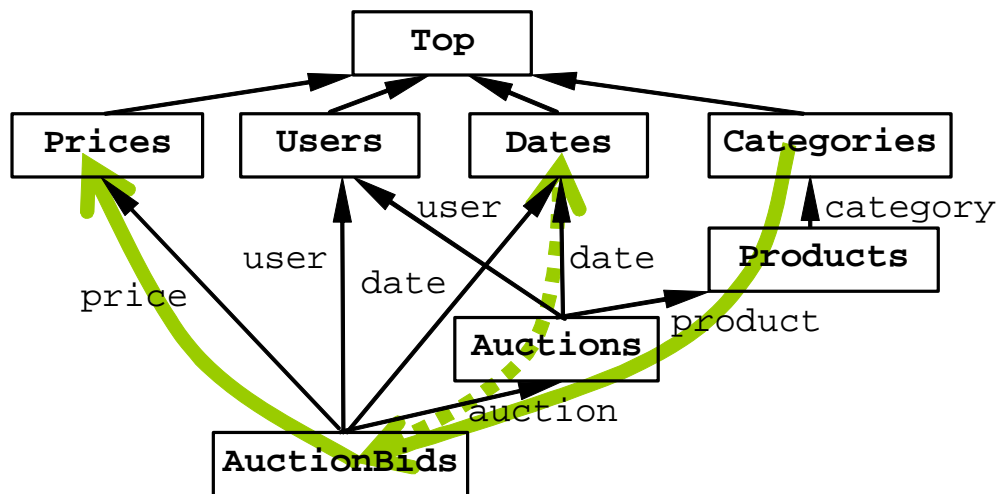
$$S = \text{Dom}(\{d\})$$



Projection and De-projection

Access path

- Access path is a sequence of projections and de-projections possibly with constraints
- Derived property is a named definition of an access path or a query
- `Category.meanPriceForTenDays = avg({ab in AuctionBids.auction.product.category | ab.auction.date > today-10 }.price);`



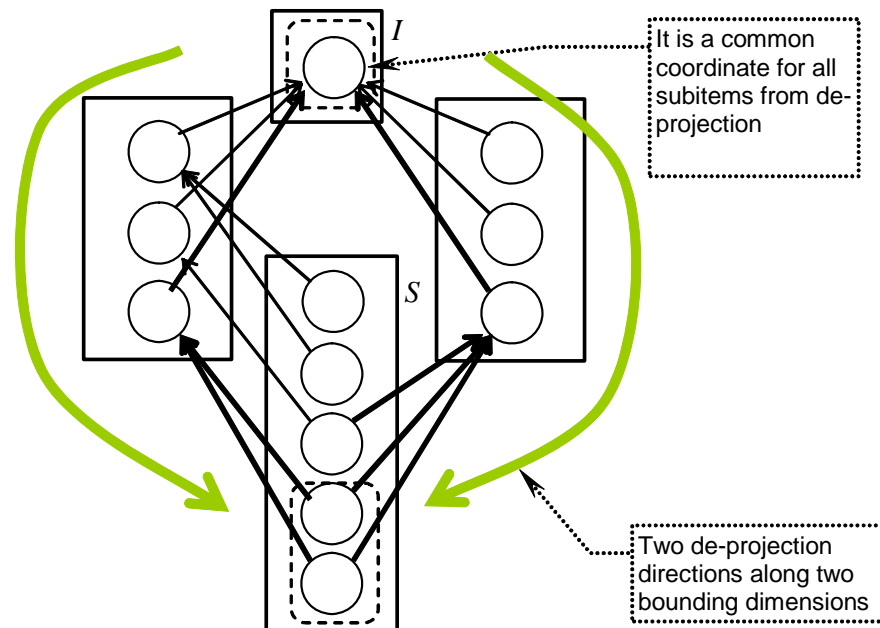
- Navigational approach with no hierarchical structure:
 - OODB
 - FDM
 - Network model

Grouping and Aggregation

Multidimensional de-projection

- More than one bounding dimension
- Multidimensional de-projection returns a set of subitems referencing source items along all bounding dimensions:

$$I \rightarrow \{d_1, d_2, \dots, d_n\} = \{s \in S \mid s.d_1 = i \wedge s.d_2 = i \wedge \dots \wedge s.d_n = i, i \in I \subseteq C\}$$

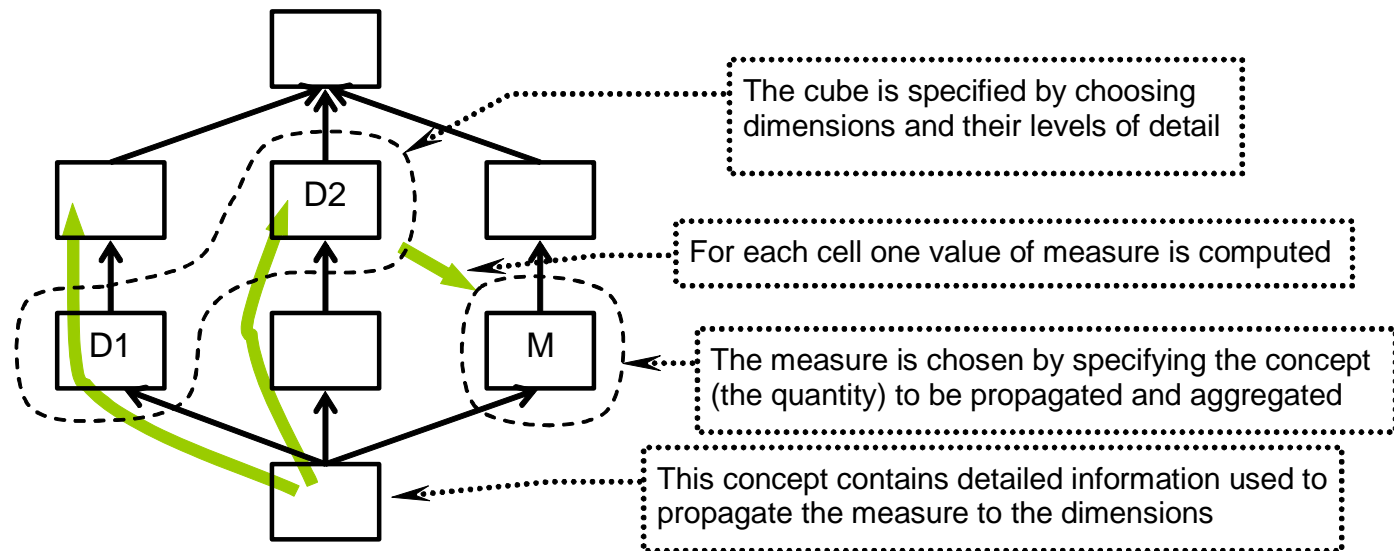


Grouping and Aggregation

Aggregation

- A dimension hierarchy is one dimension path
- Along each hierarchy we choose a concept called a level
- Universe of discourse is the Cartesian product of the chosen levels

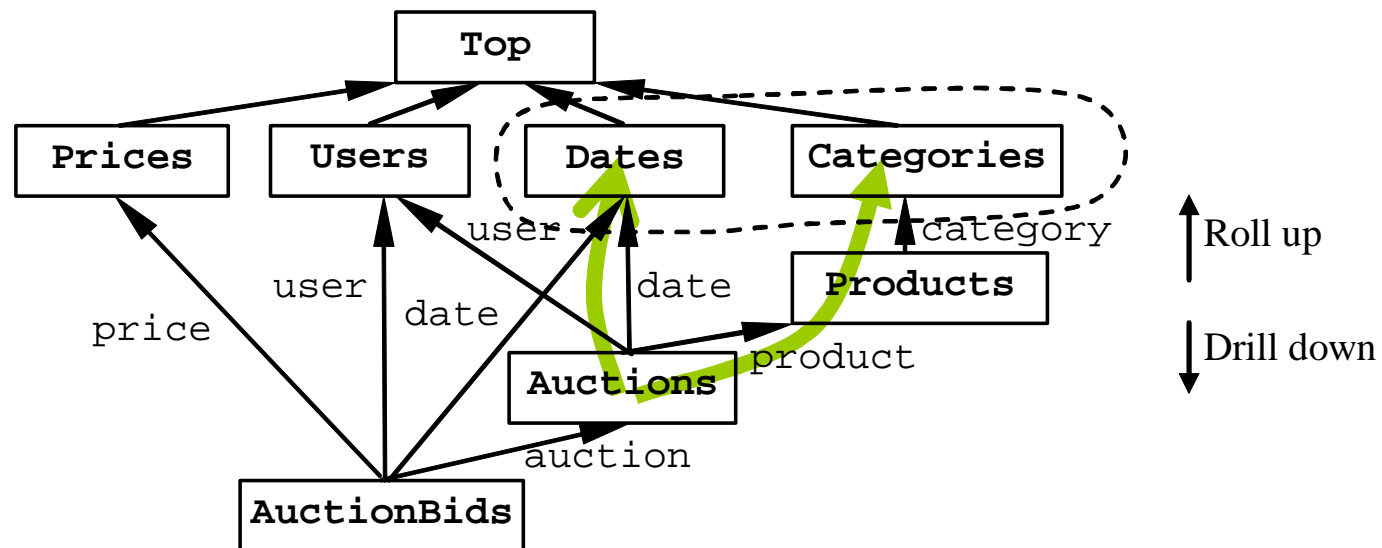
$$\Omega_L = D_1 \times D_2 \times \dots \times D_n = \{\omega = \langle \omega_1, \omega_2, \dots, \omega_n \rangle \mid \omega_j \in D_j\}$$
- For each point from UoD we find de-projection
- De-projection is aggregated



Grouping and Aggregation

Example

- $\{d : \text{Dates}, c : \text{Categories} \mid \text{isLastWeek}(d)\} < \text{avg}(\text{this} \rightarrow \{\text{Auctions.date}, \text{Auctions.product.category}\} . \text{maxBid}) \text{ as averagePrice} >$



Conclusions



- Features:
 - Global semantics
 - Hierarchical multidimensional logical structure
 - Navigation via access paths, dimensions and inverse dimensions
 - Multidimensional aggregation and analysis
 - Concept transformations (not described in this presentation)
 - Constraint propagation and inference (not described in this presentation)
- Advantages:
 - Clarity of operations
 - Easiness of use
 - Formal syntax and semantics
 - Simple query language (no joins)