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Inference in Hierarchical Multidimensional Space

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Abstract: In spite of its fundamental importance, inference has not been an inherent function of multidimensional models and analytical applications. These models are mainly aimed at numeric analysis where the notion of inference is not well defined. In this paper we define inference using only multidimensional terms like axes and coordinates as opposed to using logic-based approaches. We propose an inference procedure which is based on a novel formal setting of nested partially ordered sets with operations of projection and de-projection.

1 INTRODUCTION

Euclidean geometry is an axiomatic system which dominated for more than 2000 years until René Descartes revolutionized mathematics by developing Cartesian geometry also known as analytic geometry. The astounding success of analytical geometry was due to its ability to reason about geometric objects numerically which turned out to be more practical and intuitive in comparison with the logic-based Euclidean geometry.

The area of data modeling and analysis can also be characterized as having two branches or patterns of thought. The first one follows the Euclidean axiomatic approach where data is described using propositions, predicates, axioms, inference rules and other formal logic constructs. For example, deductive data models and the relational model are based on the first-order logic where the database is represented as a number of predicates. The second major branch in data modeling relies on the Cartesian conception where data is thought of as a set of points in a multidimensional space with properties represented as coordinates along its axes. The multidimensional approach has been proven to be extremely successful in analytical applications, data warehousing and OLAP.

Although multidimensional data models have been around for a long time (Pedersen & Jensen, 2001; Pedersen, 2009), all of them have one serious drawback in comparison with logic-based models: they do not have a mechanism of inference. Yet it is an essential function which allows for automatically deriving relevant data in one part of the model given constraints in another part without the need to specify *how* it has to be done. For example, if we need to retrieve a set of writers by specifying only a set of publishers then this could be represented by the following query:

```
GIVEN (Publishers WHERE name == 'XYZ')
GET (Writers)
```

Importantly, this query does not have any indication of what is the schema and how `Publishers` are connected with `Writers`.

Answering such queries especially in the case of complex schemas is a highly non-trivial task because multidimensional data modeling has been traditionally aimed at numeric analysis rather than reasoning. Currently existing solutions rely on data semantics (Peckham & Maryanski, 1988), inference rules in deductive databases (Ullman & Zaniolo, 1990), and structural assumptions as it is done in the universal relation model (URM) (Fagin et al., 1982; Vardi, 1988). Yet, to the best of our knowledge, no concrete attempts to exploit multidimensional space for inference have been reported in the literature, apart from some preliminary results described in (Savinov, 2006a; Savinov, 2006b).

In this paper we present a solution to the problem of inference which relies on the multidimensional structure of the database. More specifically, the paper makes the following contributions: 1) We introduce a novel formal setting for describing multidimensional spaces which is based on *nested partially ordered sets*. 2) We define operations of *projection* and *de-projections* which serve as a basis for inference. 3) We define procedures of *constraint propagation* and *inference* as well as describe how they are supported by the query language.

The paper has the following layout. Section 2 describes the formal setting by defining the notion of nested partially ordered set and how it is used to represent hierarchical multidimensional spaces. Section 3 defines main operations on nested posets and how they are used for inference. Section 4 makes concluding remarks.

2 CONCEPT-ORIENTED MODEL

The approach to inference described in this paper relies on a novel unified model, called the concept-oriented model (COM) (Savinov, 2011a; Savinov, 2011b; Savinov, 2012). One of the main principles of COM is that an element consists of two tuples: one identity tuple and one entity tuple. These identity-entity couples are modeled by a novel data modeling construct, called *concept* (hence the name of the model), which generalizes classes. Concept fields are referred to as *dimensions*. Yet, in this paper we will not distinguish between identities and entities by assuming that an element is one tuple.

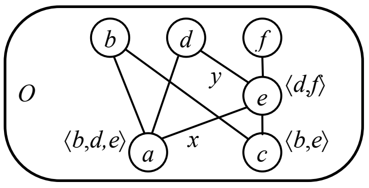


Figure 1: Database is a partially ordered set.

For this paper, another COM major principle is important which postulates that a set of data elements is a partially ordered set (poset). Another approach where posets are used for data modeling is described in (Raymond, 1996). In COM, posets are represented by tuples themselves, that is, tuple membership relation induces partial order relation '<' (less than): $\langle \ldots, e, \ldots \rangle <_1 e$. Here $<_1$ means 'immediately less than' relation ('less than' of rank 1). If $a < b$ then a is referred to as a *lesser element* and b is referred to as a *greater element*. Thus tuple members are supposed to be immediately greater than the tuples they are included in. And conversely, a tuple is immediately less than any of its member tuples it is composed of. Since tuple membership is implemented via references (which are identity tuples), this principle essentially means that an element references its greater elements. Fig. 1 is an example of a poset graphically represented using a Hasse diagram where an element is drawn under its immediate greater elements and is connected with them by edges.

At the level of concepts, tuple order principle means that dimension types specify greater concepts. Then a set of concepts is a poset where each concept has a number of greater concepts represented by its dimension types and a number of lesser concepts which use this concept in its dimensions. For example, assume that each book has one publisher:

```
CONCEPT Books // Books < Publishers
  IDENTITY
    CHAR(10) isbn
  ENTITY
    CHAR(256) title
    Publishers publisher //Greater concept
```

According to this principle, `Publishers` is a greater concept because it is specified as a type (underlined) of the dimension `publisher`.

The main benefit of using partial order is that it has many semantic interpretations: *attribute-value* (greater elements are values characterizing lesser elements), *containment* (greater elements are sets consisting of lesser elements), *specific-general* (greater elements are more general than lesser elements), *entity-relationship* (lesser elements are relationships for greater elements), and *multidimensional* (greater elements are coordinates for lesser elements). These interpretations allow us to use COM as a unified model.

In the context of this paper, the most important property of partial order is that it can be used for representing multidimensional hierarchical spaces. The basic idea is that greater elements are interpreted as coordinates with respect to their lesser elements which are interpreted as points. Thus an element is a point for its greater elements and a coordinate for its lesser elements. In Fig. 1, e is a point with coordinates d and f (its greater elements) and at the same time it is a coordinate for two points a and c (its lesser elements).

In multidimensional space, any coordinate belongs to some axis and any point belongs to some set. The question is how the notions of axes and sets can be formally represented within the order-theoretic setting. To solve this problem we assume that a poset consists of a number of subsets, called *domains*: $O = X_1 \cup X_2 \cup \ldots \cup X_m$, $X_i \cap X_j = 0$, $\forall i \neq j$. Domains are interpreted as either sets of coordinates (axes) or sets of points (spaces), and any element is included in some domain: $e \in X_k \subset O$. Domains are also partially ordered and represented by tuples so that a domain is defined as a tuple consisting of its immediate greater domains: $X = \langle X_1, X_2, \ldots, X_n \rangle$, $X <_1 X_i$.

Any element participates in two structures simultaneously: (i) it is included in some domain via the membership relation '$\in$', and (ii) it has some greater and lesser elements via the partial order relation '<'. In addition, the two structures are connected via the *type constraint*:

$$e \in D \subset O \quad \Rightarrow \quad e.x \in D.x$$

Here *e.x* is a greater element of *e* along dimension *x* and *D.x* is a greater domain of *D* along the same dimension *x*. This constraint means that an element may take its greater elements only from the greater domains. Such a set is referred to as a *nested poset*.

An example of a nested poset representing a 2-dimensional space is shown in Fig. 2. This set consists of 3 domains *X*, *Y* and *Z* where $Z = \langle X, Y \rangle$ which means that *Z* has two greater domains *X* and *Y*. Element z_1 is defined as a tuple $\langle x_1, y_1 \rangle$, that is, x_1 and y_1 are greater elements for z_1. Therefore, $z_1 \in Z$ is interpreted as a point while $x_1 \in X$ and $y_1 \in Y$ are its coordinates. According to the type constraint, elements from *Z* may take their greater elements only in *X* and *Y*.

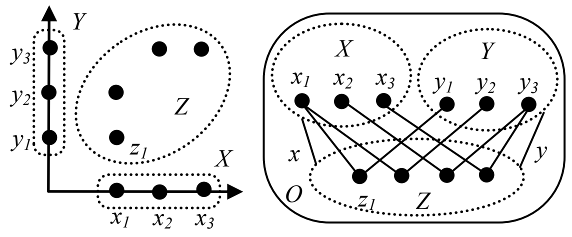


Figure 2: Nested poset representing 2-dimensional space.

A *schema* is defined as a partially ordered set of concepts where greater concepts are types of this concept dimensions. A *database schema* is defined as a partially ordered set of domains (collections) where elements within each domain have a type of some concept from the schema. A *database* is a partially ordered set of elements each belonging to one collection and having a number of greater elements referenced from its dimensions.

3 INFERENCE

3.1 Projection and De-Projection

Geometrically, projection of a set of points is a set of their coordinates. De-projection is the opposite operation which returns all points with the selected coordinates. In terms of partial order, projection means finding all greater elements and de-projection means finding all lesser elements for the selected of elements. Taking into account that greater elements are represented by references, projection is a set of elements referenced by the selected elements and de-projection is a set of elements which reference the selected elements.

To formally describe these two operation we need to select a subset Z' of points from the source domain, $Z' \subset Z$, and then choose some dimension *x* with the destination in the target greater domain $D > Z$. Then projection, denoted as $Z' \rightarrow x \rightarrow D$, returns a subset of elements $D' \subset D$ which are greater than elements from Z' along dimension *x*:

$$Z' \rightarrow x \rightarrow D = D' = \{ d \in D \mid z <_x d, z \in Z' \} \subset D$$

Note that any element can be included in projection only one time even if it has many lesser elements.

De-projection is the opposite operation. It returns all points from the target lesser domain *F* which are less than the elements from the source subset Z':

$$Z' \leftarrow x \leftarrow F = F' = \{ f \in F \mid f <_x z, z \in Z' \} \subset F$$

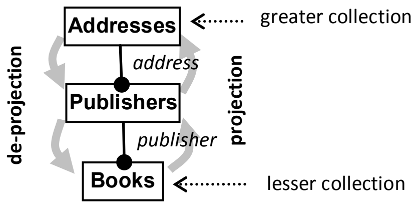


Figure 3. Projection and de-projection in schema.

In the concept-oriented query language (COQL), a set of elements is written in parentheses with constraints separated by bar symbol. For example, `(Books | price < 10)` is a set of all cheap books. Projection operation is denoted by right arrow `'->'` followed by a dimension name which is followed by the target collection. In the database schema, projection means moving up to the domain of the specified dimension (Fig. 3). It returns all (greater) elements which are referenced by the selected elements. For example (Fig. 4), all publishers of cheap books can be found by projecting them up to the `Publishers` collection along the `publisher` dimension:

```
(Books | price < 10)
  -> publisher -> (Publishers)
```

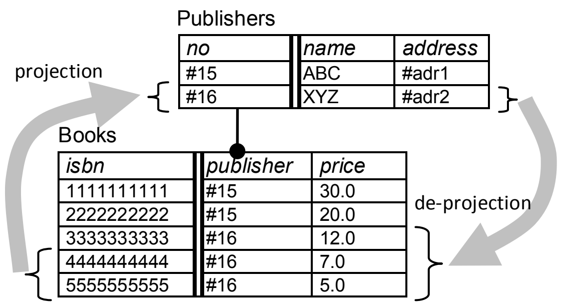
De-projection is the opposite operation denoted by left arrow `'<-'`. It returns all (lesser) elements which reference elements from the selected source collection. For example, all books published by a selected publisher can be found as follows (Fig. 4):

```
(Publishers | name=="XYZ")
  <- purchase <- (Books)
```

Projection and de-projection operations have two direct benefits: they eliminate the need in join and group-by operations. Joins are not needed because sets are connected using multidimensional hierarchical structure of the model. Group-by is not needed because any element is interpreted as a group consisting of its lesser elements. Given an element (group) we can get its members by applying de-projection operation. For example, if it is necessary to select only publishers with more than 10 books then it can be done it as follows:

```
(Publishers |
  COUNT(publisher <- (Books)) > 10)
```

Here de-projection `publisher <- (Books)` returns all books of this publisher and then their count is compared with 10.



Publishers

no	name	address
#15	ABC	#adr1
#16	XYZ	#adr2

Books

isbn	publisher	price
1111111111	#15	30.0
2222222222	#15	20.0
3333333333	#16	12.0
4444444444	#16	7.0
5555555555	#16	5.0

Figure 4. Example of projection and de-projection.

3.2 Constraint Propagation

A query can be defined as consisting of constraints and propagation rules. Constraints specify conditions for selecting elements from one domain. Propagation rules indicate how constraints imposed on one domain are used to select elements from another domain. For example, we can select a book title, publishing date or price while the task is to find all publishers which satisfy these selected values. Since source constraints and target elements belong to different parts of the model, propagation rules are needed to connect them.

The main benefit of having projection and de-projection operations is that they allow us to easily propagate arbitrary constraints through the model by moving up (projection) and down (de-projection). For example, given a collection of `Books` we can find all related addresses by projecting it up to the `Addresses` collection (Fig. 3):

```
(Books | price < 10)
  -> publisher -> (Publishers)
  -> address -> (Addresses)
```

In most cases either intermediate dimensions or collections can be omitted by allowing for more concise queries:

```
(Books | price < 10)
  -> publisher -> address

(Books | price < 10)
  -> (Publishers) -> (Addresses)
```

De-projection allows us to move down through the partially ordered structure of the model which is interpreted as finding group members. For example, given a country code we can find all related `Books` using the following query (Fig. 3):

```
(Addresses | country == 'DE')
  <- address <- (Publishers)
  <- publisher <- (Books)

(Addresses | country == 'DE')
  <- (Publishers) <- (Books)
```

Note that such queries are very useful for nested grouping which are rather difficult to describe using the conventional group-by operator.

Constraint propagation can be further simplified if instead of a concrete dimension path we specify only source and target collections. The system then reconstructs the propagation path itself. Such projection and de-projection with an undefined dimension path will be denoted by `'*->'` and `'<-*'` (with star symbol interpreted as *any* dimension path). The previous two queries can be then rewritten as follows:

```
(Books | price < 10) *-> (Addresses)

(Addresses | country == 'DE') <-* (Books)
```

3.3 Inference

Automatically propagating constraints only up or down is a restricted version of inference because only more general (projection) or more specific (de-projection) data can be derived. If source and target collections have arbitrary positions in the schema then this approach does not work because they do not belong to one dimension path that can be used for projecting or de-projecting. In the general case, constraint propagation path consists of more than one projection and de-projection steps. Of course, this propagation path can be specified explicitly as part of the query but our goal is to develop an *automatic* procedure for finding related items in the database which is called inference.

The main idea of the proposed solution is that source and target collections have some common lesser collection which is treated as *dependency* between them. Such common lesser collections are also used to represent relationships between their greater collections as opposed to the explicit use of relationships in the entity-relationship model (Savinov, 2012). Constraints imposed on the source collection can be used to select a subset of elements from this lesser collection using de-projection. And then the selected elements are used to constrain the target elements using projection. In OLAP terms, we impose constraints on dimension tables, propagate them down to the fact table, and then finally use these facts to select values from the target dimension table. In terms of multidimensional space this procedure means that we select points along one axis, then de-project them to the plane by selecting a subset of points, and finally project these points to the target axis by using these coordinates as the result of inference.

If X and Y are two greater collections, and Z is their common lesser collection then the proposed inference procedure consists of two steps:

1. [De-projection] Source constraints $X' \subset X$ are propagated down to the set Z using de-projection: $Z' = X' \leftarrow * \leftarrow Z \subset Z$
2. [Projection] The constrained set $Z' \subset Z$ is propagated up to the target set Y using projection: $Y' = Z' \rightarrow * \rightarrow Y \subset Y$

Here by star symbol we denote an arbitrary dimension path. In the case of n independent source constraints $X_1', X_2', \ldots, X_n'$ imposed on sets $X_1, X_2, \ldots, X_n$ the de-projection step is computed as an intersection of individual de-projections: $Z' = \bigcap X_i' \leftarrow * \leftarrow Z$.

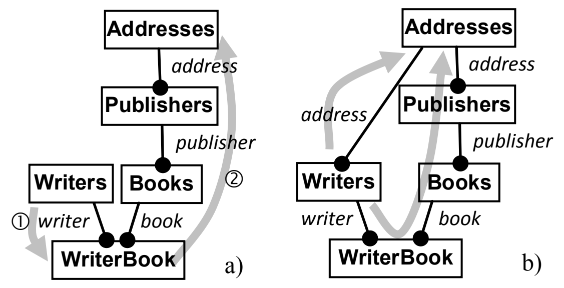


Figure 5. Inference via de-projection and projection.

In COQL, inference operator is denoted as `'<-*->'` (de-projection step followed by projection step via an arbitrary dimension path). It connects two collections from the database and finds elements of the second collection which are related to the first one. To infer the result, the system chooses their common lesser collection and then builds de-projection and projection dimensions paths. After that, inference is performed by propagating source constraints to the target along this path. For example (Fig. 5a), given a set of young writers we can easily find related countries by using only one operator:

```
(Writers | age < 30)
  <-*-> (Addresses) -> countries
```

To answer this query, the system first chooses a common lesser collection, which is `WriterBooks` in this example, and then transforms this query to two operations of de-projection and projection:

```
(Writers | age < 30)
  <-* (WriterBooks) // De-project
  *-> (Addresses) -> countries // Project
```

After that, the system reconstructs the complete constraint propagation path:

```
(Writers | age < 30)
  <- writer <- (WriterBooks)
  -> book -> (Books)
  -> publisher -> (Publishers)
  -> address -> (Addresses) -> countries
```

In the case of many dependencies (common lesser collections) or many de-projection/projection paths between them, the system propagates constraints using all of them. This means that all source constraints are first propagated down along all paths to all lesser collections using de-projection. After that, all the results are propagated up to the target collection using all existing dimension paths.

If the user wants to customize inference and use only specific dimensions or collections then they can be provided as part of the query. For example, assume that both `Publishers` and `Writers` have addresses (Fig. 5b). Accordingly, there are two alternative paths from the source to the target and two alternative interpretations of the relationship: writers living in some country or writers publishing in this country. This ambiguity can be explicitly resolved in the query by specifying the required common collection to be used for inference:

```
(Addresses | country == 'DE')
  <-* (WriterBooks) *-> (Writers)
```

In this way, we can solve the problem of having multiple propagation paths. In the next section we consider the problem of having no propagation path between source and target collections.

3.4 Use of Background Knowledge

If the model has a bottom collection which is less than any other collection then inference is always possible because it connects *any* pair of source and target collections. The question is how to carry out inference in the case the bottom collection is absent. Formally, collections which do not have a common lesser collection are independent, that is, their elements are unrelated.

For example (Fig. 6), if books are being sold in different shops then the model has two bottom collections: `WriterBooks` and `Sellers`. Assume now that it is necessary to find all shops related to a set of writers:

```
(Writers | age < 30) <-*-> (Shops)
```

The propagation path should go through a common lesser collection which is absent in this example and therefore inference is not possible.

One solution to this problem is to formally introduce a bottom collection which is equal to the Cartesian product of its immediate greater collections. In COQL, this operation is written as a sequence of collections in parentheses separated by comma:

```
Bottom = (WriterBooks, Sellers)
```

However, this artificial bottom collection (shown as a dashed rectangle in Fig. 6) does not impose any constraints and hence `Writers` and `Shops` are still independent.

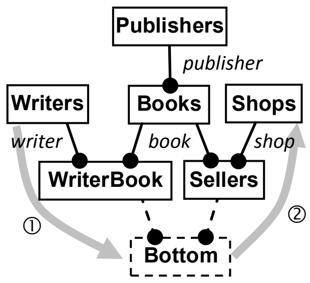


Figure 6. Use of background knowledge.

To get meaningful results we have to impose additional constraints on the bottom collection. These constraints represent implicit dependencies between data elements, called background knowledge. They can be expressed via any condition which selects a subset of elements from the bottom collection, for instance, as a dependency between its attributes. In our example, we assume that a written book is the same as a sold book:

```
Bottom = (WriterBooks wb, Sellers s |
  wb.book == s.book)
```

Now the `Bottom` collection contains only a subset of the Cartesian product of its two greater collections and can be used for inference. We simply specify this bottom collection as part of the query:

```
(Writers | age < 30)
  <-* (WriterBooks bw, Sellers s |
    bw.book == s.book)
  *-> (Shops)
```

Here the selected writers are de-projected down to the bottom collection. Then this constrained bottom collection is propagated up to the target. As a result, we will get all shops selling books written by the selected authors. Note how simple this query is especially in comparison with its SQL equivalent which has to contains many joins and explicit intermediate tables. What is more important, it is very natural because we specify what we want to get rather than how the result set has to be built.

4 CONCLUSION

In this paper we have described the idea of having inference capabilities as an inherent part of multidimensional data models and analytical query languages. The proposed approach is very simple and natural in comparison to logic-based approaches because it relies on only what is already in the database: dimensions and data. Its main benefit is that now inference can be made integral part of multidimensional databases by allowing not only doing complex numeric analysis but also performing tasks which have always been a prerogative of logic-based models.

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