SOME PROPERTIES OF NEW RESOLUTION RULE IN THE LOGIC OF POSSIBILITY DISTRIBUTIONS

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Abstract: In the paper new resolution principle and some its properties are considered. The new resolution rule is applied to any two disjuncts on some variable and results in a third disjunct called resolvent. Disjuncts in the logic of possibility distributions consist of explicitly represented possibility distributions over sets of values of variables. The property of adjacency of two disjuncts allows to find out if the resolvent is their consequence.

I. Introduction

Let us suppose that some problem domain is described by a number of attributes each of them taking a finite number of values. All combinations of values form a space of states. Our knowledge about the problem domain consists in that we prohibit some states, i.e., we say that these states are impossible or their descriptions are meaningless.

Such an approach is called the attribute model of problem domain. It originates from the works of A.D. Zakrevsky [7,8] where the technique of sectional boolean vectors was used for representing and manipulating disjuncts. Later in papers [2–6] the formalism of Zakrevsky including the technique of sectional vectors and the resolution rule was generalized on fuzzy case. In this paper we develop an approach called the logic of possibility distributions which continues this direction and consider more closely a new resolution rule proposed in the framework of this logic and some its properties.

In section II we consider elementary propositions and disjuncts of the logic of possibility distributions and their interpretation. In section III we give a general definition of the resolution rule. In section IV we discuss the property of adjacency of two disjuncts on some variable and formulate a criterion of adjacency. This property is used in inference process since it allows us to find out if the resolvent is a consequence of two its premises.

II. Logic of Possibility Distributions

Let $x_1, x_2, \ldots, x_n$ be elementary logical variables, each of them taking its values from the finite sets $X_1, X_2, \ldots, X_n$ called (elementary) domains, respectively. The domain $X_i = \{0, 1, \ldots, n_i - 1\}$ consists of n_i values. All domains are supposed to be crisp sets.

The Cartesian product of all domains $X_1 \times X_2 \times \ldots \times X_n$ forms the universe Ω with the power $n_1 \times n_2 \times \ldots \times n_n$. Each element $\omega = \langle x_1, x_2, \ldots, x_n \rangle \in \Omega$ is an ordered n-tuple of the values of all variables. Since all the domains are crisp sets, the universe is also a crisp set. The universe Ω can be viewed as a set of all states of the world, system or process to be modeled.

If there is not any knowledge about the world, then any state is absolutely possible. In the logic of possibility distributions it means that all states ω are assigned equal degree of possibility 1. When there is some knowledge, some states are less possible than others and therefore their degrees of possibility are less than 1. If the state is absolutely impossible, i.e., it is prohibited, it has the degree of possibility 0.

In the logic of possibility distributions semantics or knowledge is thought of as a possibility distribution over the universe $\pi(\omega) : \Omega \to [0, 1]$. The semantics is represented by means of a number of disjuncts. Each disjunct

$$\phi = \phi_0 \vee \phi_1(x_1) \vee \ldots \vee \phi_n(x_n)$$

consists of n elementary propositions $\phi_i(x_i)$ $(i = 1, 2, \ldots, n)$ which are represented by possibility distributions over the domains $\phi_i(x_i) : X_i \to [0, 1]$ and one constant proposition $\phi_0 \in [0, 1]$. Disjuncts are interpreted in the following way:

$$\phi(\omega) = \max\Big(\phi_0, \phi_1(x_1), \ldots, \phi_n(x_n)\Big)$$

where $\omega = \langle x_1, x_2, \ldots, x_n \rangle$.

Example 1 The problem domain is described by two attributes HEIGHT with the values from {Low, Medium, High} and WEIGHT with the values from {Small, Big}. The universe for it consists of 6 objects (Fig. 1). Some knowledge expressed by the disjunct

$$\text{WEIGHT} = \{\text{Small} : 1, \text{Big} : 0\} \vee \text{HEIGHT} = \{\text{ Low} : 0, \text{Medium} : 0.5, \text{High} : 1 \}$$

is shown in Fig. 1.

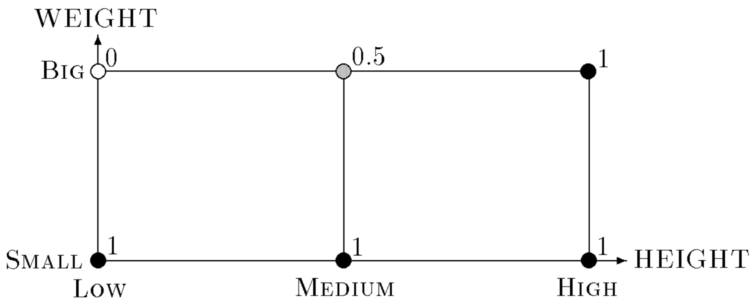

Figure 1: The universe formed from two attributes

III. Resolution Rule

The resolution rule used in the logic of possibility distributions is applied to any two disjuncts

$$\phi = \phi_0 \vee \phi_1(x_1) \vee \ldots \vee \phi_n(x_n)$$

and

$$\psi = \psi_0 \vee \psi_1(x_1) \vee \ldots \vee \psi_n(x_n)$$

on some variable x_k and results in a third disjunct

$$\mu = \phi\langle x_k \rangle\psi$$

Disjuncts which the resolution is applied to must have the constant proposition which is equal to 0. It is easy to show that it is not a restriction, because any disjunct can be transformed to such a form.

The resolvent is constructed from the source disjuncts in the following way:

$$\mu_i = \begin{cases} 0 & \text{if } i = 0 \\ \phi_i \wedge \psi_i & \text{if } i = k \\ \phi_i \vee \psi_i & \text{if } i \neq 0, i \neq k \end{cases}$$

It means that

(i) the resolvent constant proposition is always equal to 0

(ii) kth proposition of the resolvent (which the resolution is applied on) is equal to the conjunction of the two corresponding propositions from the source disjuncts

(iii) every non-kth proposition of the resolvent is equal to the disjunction of the two corresponding propositions

This rule can be represented also in the form of the following inference pattern:

$$\langle x_k \rangle \frac{\begin{array}{ccccccccc} \{0\} & \vee & \phi_1 & \vee \ldots \vee & \phi_k & \vee \ldots \vee & \phi_n \\ \{0\} & \vee & \psi_1 & \vee \ldots \vee & \psi_k & \vee \ldots \vee & \psi_n \end{array}}{\begin{array}{ccccccccc} \{0\} & \vee & (\phi_1 \vee \psi_1) & \vee \ldots \vee & (\phi_k \wedge \psi_k) & \vee \ldots \vee & (\phi_n \vee \psi_n) \end{array}}$$

where elementary propositions with the same number are arranged in columns.

It is proven in [6] that the resolvent is a consequence of two premises:

$$\text{if } \mu = \phi\langle x_k \rangle\psi, \quad \text{then } \phi \wedge \psi \models \mu$$

Example 2 The following example shows that this resolution rule can be viewed as a generalized *modus ponens* for the logic of possibility distributions:

$\langle WEIGHT \rangle$

	WEIGHT		HEIGHT			
	Small	Big	Low	Medium	High	
φ	1	0	0	0.5	1	disjunct 1
ψ	0	1	0	0	0	disjunct 2
μ	0	0	0	0.5	1	resolvent

The first disjunct represents a dependency between two attributes, while the second disjunct and the resolvent represent facts, i.e., propositions about one variable.

IV. Adjacency of Disjuncts

Two disjuncts ϕ and ψ are said to be adjacent on the variable x_k if they satisfy the following condition:

$$\phi \not\models \phi\langle x_k \rangle\psi \quad \text{and} \quad \psi \not\models \phi\langle x_k \rangle\psi$$

where $\phi\langle x_k \rangle\psi$ is their resolvent on kth variable. If disjuncts are not adjacent, then the resolvent is a consequence of one of them, and hence it does not contain new information.

Example 3 The following two disjuncts are adjacent on the first variable and they are not adjacent on the second variable:

$$\langle x_1 \rangle \frac{\begin{array}{rcr} \{1, 0.5, 0.2\}(x_1) & \vee & \{1, 0\}(x_2) \\ \{0, 0.2, 1\}(x_1) & \vee & \{0, 0\}(x_2) \end{array}}{\begin{array}{rcr} \{0, 0.2, 0.2\}(x_1) & \vee & \{1, 0\}(x_2) \end{array}} \qquad \langle x_1 \rangle \frac{\begin{array}{rcr} \{1, 0.5, 0\}(x_1) & \vee & \{1, 0.2\}(x_2) \\ \{0, 0.2, 1\}(x_1) & \vee & \{0, 0\}(x_2) \end{array}}{\begin{array}{rcr} \{1, 0.5, 1\}(x_1) & \vee & \{0, 0\}(x_2) \end{array}}$$

Since the resolution is used in inference process it is important to have a criterion by which we could find out if two disjuncts are adjacent on some variable or they are not. Otherwise the only way is to build resolvents for all disjunct pairs and on all variables. To formulate such a criterion we will need two characteristics of elementary propositions.

The minimal value that the possibility distribution of the proposition takes over its domain is said to be constant of the proposition:

$$\text{constant}(\phi) = \min_{x \in X} \phi(x), \quad \text{where } \phi(x) : X \to [0, 1]$$

The degree of incomparability of the proposition ϕ in relation to ψ is equal to the maximal possibility distribution value of the proposition ϕ which is exactly greater than the corresponding (i.e., in the same point of the domain) value of the proposition ψ:

$$\text{incomp}_{\psi}(\phi) = \max_{\phi(x) > \psi(x)} \phi(x), \quad \text{where } \ x \in X$$

Thus in order to compute this quantity first it is necessary to select in the proposition ϕ all the possibility distribution values which are exactly greater than the corresponding values in ψ (they are decreased when conjuncting with the components from ψ), and then to choose among them the maximal value (Fig. 2). In the case the condition $\forall x \in X \ \phi(x) \leq \psi(x)$ holds, i.e., there is nothing to choose the maximal value from, it is supposed by definition that $\text{incomp}_{\psi}(\phi) = 0$.

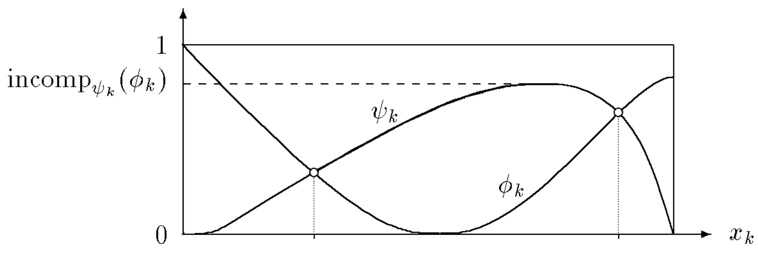

Figure 2: Relative degree of incomparability

Example 4 Degree of incomparability of the proposition $\phi_i = \{0, 0.5, 0.7, 1\}(x_i)$ in relation to the proposition $\phi_i = \{0.2, 0.4, 0.6, 1\}(x_i)$ is equal to $\text{incomp}_{\psi_i}(\phi_i) = \max(0.5, 0.7) = 0.7$, whereas $\text{incomp}_{\phi_i}(\psi_i) = \max(0.2) = 0.2$.

The (mutual) degree of incomparability is equal to the minimal of two relative degrees of incomparability:

$$\text{incomp}(\phi, \psi) = \min\Big(\text{incomp}_{\psi}(\phi), \text{incomp}_{\phi}(\psi)\Big)$$

The criterion of adjacency of two disjuncts on some variable is formulated in the following theorem.

Theorem Disjuncts ϕ and ψ are adjacent on the variable x_k iff

$$\forall i \neq k \quad \text{constant}(\phi_k \vee \psi_k) < \text{incomp}(\phi_k, \psi_k)$$

There are two factors which affect the adjacency of two disjuncts:

1. The degree of incomparability $\text{incomp}(\phi_k, \psi_k)$. If this value is too low, then the disjuncts are not adjacent. In particular, if one of propositions ϕ_k and ψ_k is included into the other ($\text{incomp}(\phi_k, \psi_k) = 0$), the disjuncts are not adjacent. This factor is a generalization of the conventional condition (see, e.g., [1]) that the resolution can be applied only to disjuncts involving contrary literals.
2. The constant of non-kth propositions of the resolvent. If one of these $n - 1$ values is too high, then the disjuncts are not adjacent. This factor is a generalization of the conventional condition which consists in the absence in disjuncts of the second pair of contrary literals.

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